The First Conference on Information Theory and Complex Systems
TINKOS 2013

BOOK OF ABSTRACTS

Editors: Velimir Ilić and Miomir Stanković

Belgrade, Serbia, September 25, 2013
Mathematical Institute of the Serbian Academy of Sciences and Arts

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Република Србија
МИНИСТАРСТВО ПРОСВЕТЕ
НАУКЕ И ТЕХНОЛОШКОГ
РАЗВОЈА

The conference is organized by
the Mathematical Institute of the Serbian Academy of Sciences and Arts
under auspices of
the Ministry of Education, Science and Technology Development
of the Republic of Serbia

CONFERENCE PROGRAM

OPENING	
10:00–10:10	Miomir Stanković
SESSION 1	Information and complexity measures
	Chairman – Milan Rajković
10:10–10:30	Velimir Ilić, Miomir Stanković – On the characterization and relationship between q-additive information measures and certainty measures
10:30–10:50	Edin Mulalić, Miomir Stanković, Radomir Stanković – Notes on evolution of axiomatic characterization of the Tsallis entropy
10:50–11:10	Marija Boričić – Entropy and Ergodic Theory
11:10–11:30	Miloš Milovanović, Milan Rajković – Complexity Measure for Representation Optimality
11:30–11:50	Stanislav Stanković – Computing Estimation of Entropy of Binary Sequences by Using Decision Diagrams
11:50–12:10	Miloš Djurić – On the Dempster-Shafer Theory of Evidence and Some Modifications of Dempster's Rule of Combination
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13:20–13:40	Milan Todorović, Bojan Marinković, Aleksandar Zeljić, Paola Glavan, Zoran Ognjanović – Formal Description of the Chord Protocol using Isabelle/HOL Proof Assistant
13:40–14:00	Bojan Marinković, Vincenzo Ciancaglini, Zoran Ognjanović, Paola Glavan, Luigi Liquori, Petar Maksimović – Analyzing the Exhaustiveness of the Synapse Protocol
14:00–14:20	Aleksandar Janjić, Suzana Savić, Lazar Velimirović – Development of Energy System Smart Grids Based on Multi-Criteria Decision Making
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17:20–17:40 Suzana Savić, Miomir Stanković , Goran Janaćković – Fuzzy AHP Ranking of Occupational Safety System Quality Indicators

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SESSION 1
Information and complexity measures

Chairman – Milan Rajković

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On the characterization and relationship between q-additive information measures and certainty measures

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Notes on evolution of axiomatic characterization of the Tsallis entropy

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Complexity Measure for Representation Optimality

Stanislav Stanković
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Miloš Djurić
On the Dempster-Shafer Theory of Evidence and Some Modifications of Dempster's Rule of Combination

On the characterization and connection between q-additive information measures and multiplicative certainty measures

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Keywords

information measure; entropy; certainty; axiomatic characterization; q-addition

Summary

In past decades, information theory and physics communities have shown a plausible interest in the characterization of information and certainty measures. The measures have been defined in different ways and with different motivations, but all of them share certain basic properties [3].

The basic measure in information theory is the Shannon entropy [8] and is defined as linear (trace-form) expectation of an additive decreasing function of an event probability called information content. Renýi [7] and Varma [12] generalizes the Shannon entropy with the additive information measure which can be represented as quasi-linear mean of the information content. Havrda and Charvát [2] and Tsallis [10] consider the entropies which are the trace form of q-additive [4] information content. The class of entropies which are quasi-linear mean of the q-additive information are considered by Sharma and Mittal [9].

Certainty measures are defined as the average value of a multiplicative increasing function of the event probability called certainty content. The certainty measures which can be represented as the trace form expectation of the certainty content are Onicesu's information energy [5] and order-α weighted information energy introduced by Pardo [6]. The certainty measures which can be represented as the quasi-linear expectation are considered by Lubbe [11] and Bhatia [1].

We characterizes these measures in unique way starting from a set of axioms which are based on common properties for all of them. According the axiomatic system, compassable information/certainty measure is represented as quasi-linear mean-value of compassable information/certainty content. The composition operation is defined using the monotonic function $h : R \to R$, which is monotonically increasing for the information and monotonically decreasing for the certainty measures.

Particularly, we pay attention to the most common measures appearing in literature, generated by the polynomial combination operation and define the polynomial information and certainty measures. It is shown that q-addition and real product represents the unique polynomial combinations operations which ensure that the information content is decreasing and the certainty content is increasing.

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Notes on evolution of axiomatic characterization of the Tsallis entropy

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Keywords

Tsallis entropy; axiomatization; trace-form; additivity; non-extensive entropy

Summary

The Tsallis entropy was proposed as a possible generalization of the standard Boltzmann-Gibbs-Shannon (BGS) entropy as a concept aimed at efficient characterisation of non-extensive complex systems. Ever since its introduction [1], it has been successfully applied in various fields [2]. In parallel, there have been numerous attempts to provide its formal derivation from an axiomatic foundation, for example [3, 4, 5, 6, 7, 8, 9]. Since the Tsallis entropy can be considered as a one-parameter generalization of the Shannon entropy in the sense that it reduces to the Shannon entropy in limiting case, one way to attack the problem of its axiomatization is by generalizing one of axiomatic systems developed for the Shannon entropy. However, it is not always obvious which is the most appropriate or natural way to do it. For example, different generalizations of the Shannon-Khinchin axioms have been proposed by at least three researchers [3, 4, 6]. Various other approaches to the problem of axiomatization of the Tsallis entropy have been explored and the topic continues to be the subject of debate in the scientific community. The debate is not constrained to advantages and disadvantages of a particular axiomatization, but questions about completeness and correctness also have been raised. Given the sheer number of the proposed axiomatizations, as well as mentioned issues, it is not an easy task to comprehend the current state of this topic. This is not surprising considering the fact that the Tsallis entropy itself has been rediscovered over times, independently by several researchers, as noted by Tsallis in [10].

In this talk, we present a brief overview of a class of axiomatic systems purposely developed for the characterization of the Tsallis entropy and investigate motivation for introducing each of them. We explore relationships among them, and show how one axiomatic system led to another, building in this way a chronological map of reappearances of definitions the Tsallis entropy and its various characterizations in scientific literature. In addition, we will discuss some of the existing issues and propose possible solutions, which can be viewed as a contribution to the problem of axiomatization of the Tsallis entropy [6, 7, 11]. In analysis of various axiomatic systems, certain concepts which require special attention and discussion are observed as particularly important. Accordingly, in these discussions we emphasize the role of the following concepts:

- *Additivity* – In which way the entropy of a composite system can be expressed in terms of entropies of its subsystems? So far, various types of additivity have been utilized, for example, the pseudo-additivity, generalized Shannon additivity and generalized Fadeev additivity.
- *Dependency* – How dependency of two systems affects their composition? Some axiomatizations exploit composition of independent systems, while others do not make such restrictions.
- *Trace-form* – Is the trace-form property taken as an assumption (an axiom) or does it follow as a consequence?

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Entropy and Ergodic Theory

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Keywords

entropy; ergodic theory; logical system; partition; dynamical system.

Summary

Since the beginning of the nineteenth century, when the first needs to introduce a measure of stochasticity of given system emerged, until today, when almost each system requires the determination of the measure of its vagueness, the concept of entropy has been evolving. Entropy, as one of the basic notions in information theory, roughly defined as measure of uncertainty in a random variable, after appearing in thermodynamics (Boltzmann's entropy), naturally appears in information theory (Shannon's entropy) and measure theory, and finally in theory of dynamical systems (Kolmogorov–Sinai entropy) [2]. Ergodic theory, which studies dynamical systems with invariant measure, enables defining concept of entropy, completely different from the one which is typical for stochastic systems. Entropy in deterministic systems will be our main subject. The evolution of the notion of entropy is under our scope, as well as some theorems concerning ergodic-theoretic entropy.

One of the major questions of ergodic theory is the classification of dynamical systems [5,6,7], i.e. whether two systems are isomorphic to one another. Attaching isomorphism invariants to system, such as ergodicity, weak and strong mixing, periodicity, helps answering the previous question. The entropy of a dynamical system is an example of invariant, so it is used for system classification. We present one way of logical systems classification regarding their entropies.

The concept of generalized Shannon's entropy – entropy of a partition and the logical system represented by its Linednbaum-Tarski algebra makes it possible to define the entropy of a many-valued propositional logic [1,3,4]. Our finite measure of uncertainty *H* of a finite-valued logic monotonically increases with the growth of truth values number. This measure is sensitive on both the number *m* of truth values of an *m*-valued logic, and on the number *k* of its designated (true) values. The definition of a logical system entropy we proposed, allows to classify finite-valued propositional logics.

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Complexity Measure for Representation Optimality

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Keywords

Self-organized systems; Markov processes; structures and organization in complex systems

Summary

Since there have been only a few studies on the choice of the optimal wavelet basis in signal processing reported up until now [1], there is no comprehensive and systematic study that addresses this issue. Recently, we have studied this problem from the aspect of self-organization [2] based on the multifold and multidisciplinary approach to the analysis of temporal or spatiotemporal data, which resulted in the resolvment of several important data-related problems simultaneously. Namely, the method enables determination of the optimal wavelet basis, quantifies statistical complexity and self-organization of the system under study and performs superior noise reduction. Additionaly, the method offers new perspective on prediction using the concepts of information theory applied to self-organizing phenomena.

The wavelet transform decomposes the signal in terms of the shifted and dilated versions of the prototype bandpass wavelet function (mother wavelet) and shifted versions of the lowpass scaling function [3]. The wavelet decomposition tends to be sparse in the sense that most of the signal energy is distributed into small number of large (yang) coefficients while the rest of the energy is dispersed in the small (yin) coefficients. The distribution of the wavelet coefficients density can be modeled by a mixture of two Gaussians, with each component corresponding to the yin and yang states of the coefficients. Thus to each coefficient corresponds hidden (unknown) yin or yang state, and the dependency between hidden states within and across resolution levels is modeled by the Hidden Markov model [4]. The hidden states are considered as causal states in the sense that they show how one state of affairs leads to another, future one.

The complexity of the system is identified with an amount of information needed to specify its causal state. The local statistical complexity is defined as the entropy of local causal state $C(x,t) = H(S(x,t))$ and the optimal basis is chosen as the one which maximizes the global complexity defined as $C = H(S)$, where S is the joint distribution of all hidden variables of the wavelet tree.

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Computing Estimation of Entropy of Binary Sequences by Using Decision Diagrams

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Keywords

Entropy estimation; Decision trees; Decision diagrams; Binary sequences

Summary

The concept of entropy as defined by Shannon [1] is a fundamental concept in information theory since describes the amount of information carried by a signal and it applicability greatly transcends the field of information theory in a narrow sense. The estimation of entropy is an important task and there is a variety of methods to perform it. We are discussing the method introduced in [2] defined as follows.

Consider an alphabet A and the set A^* of all finite words over A. For a given vector $f = f_1 f_2 \cdots f_t \in A^*$ of length t, we consider a possible subvector $v = v_1, v_2, \cdots, v_k$, $k \leq t$. The rate of occurrences of v in f is denoted as $\nu_f(v)$.

For any $0 \leq k \leq t$, the empirical Shannon entropy of order k is estimated as [2]

$$h_k^* = -\sum_{v \in A^k} \frac{\overline{\nu}_f(v)}{(t-k)} \sum_{a \in A} \frac{\nu_f(va)}{\overline{\nu}_f(v)} \log \frac{\nu_f(va)}{\overline{\nu}_f(v)}.$$

By using this expression, to calculate the entropy estimate, it is necessary to determine the number of occurrences of all possible subvectors v of length k in f. This can be done by a brute force method by a moving window of length k over f and increasing an appropriate counter for each encountered subvector. This is, however, a computationally demanding task. The amount of computations can be reduced if the underlying data structure is selected such that properties of the binary sequence f are taken into account. Decision diagrams for the representation of binary sequences f of length equal to a power of 2 are a data structure in which repeated identical subvectors are removed. Therefore, decision diagrams are a suitable data structure for computing the entropy estimates as defined above. This is a motivation to discuss in this paper a method for computing the entropy estimates of binary sequences of length $t = 2^n$ for $k = 2$. The method is based on the following observations. The left and the right part of a given f are $f_{s_1} = f_1 f_2 \cdots f_{\frac{t}{2}}$ and $f_{s_2} = f_{\frac{t}{2}+1} f_{\frac{t}{2}+2} \cdots f_t$. Then, obviously, the number of occurrences of a subvector v of length $k = 2$ is

$$\nu_f(v) = \nu_{f_{s_1}}(v) + \nu_{f_{s_2}}(v) + c,$$

where c takes into account the case when v occurs exactly at the split of two halves of f so that its first character is in f_{s1} while the second is in f_{s2}. Thus, if $v = v_1 v_2$, then

$$c = \begin{cases} 1, & \text{if } v_1 = f_{\frac{t}{2}} \text{ and } v_2 = f_{\frac{t}{2}+1}, \\ 0, & otherwise. \end{cases}$$

This observation can be recursively applied by splitting f into shorter and shorter subvectors until subvectors of length 2. Whenever a subvector v identical to a particular segment of f is noticed, $\nu_f(v)$ is increased for 1. This leads to a recursive procedure that resembles the recursive structure of a binary decision tree. Binary decision diagrams (BDD) are derived by reduction of binary decision trees by removing identical subvectors in sequences to be represented. The reduction rules are defined such that the BDD for a given f contains the complete information about f. Therefore, the procedure for computing the entropy estimate can be implemented over the BDD. This allows to avoid repeated computations and ensures the efficiency of the procedure.

This method was originally presented in [3] and also further discussed in [4].

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On Dempster-Shafer Theory of Evidence and Some Modifications of Dempster's Rule of Combination

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Keywords

Dempster-Shafer theory; theory of evidence; Dempster's rule of combination; Bayes' theorem; conflicts in theory of evidence

Summary

One of the problems of standard probability approaches is the absence of any quantification for confidence in our beliefs. For instance, let's consider a coin where we don't know if it is fair or not (it can even have heads on both sides). If we know nothing about this coin, a Bayesian approach would assign the prior probabilities P(Head)=P(Tail)=0.5. So it is clear that a single probability in Bayesian approach does not capture how uncertain we are about first-step prior probabilities. Insufficient precision may also occur in a case of so called subjective uncertainty due incomplete set of data, measurement error or subjective interpretation of available data.

An approach to addressing this problem is to consider an interval of probabilities, specifying reasonable upper and lower bounds on what is reasonable to believe the probability lies. For example, if experimental coin has never been flipped and so we know nothing about it, our belief about the coin coming up heads might be between [0, 1]. In contrast, after flipping the coin theoretically infinite number of times and so finding it fair, our belief in the coin coming up heads would be close to [0.5,0.5].

One of the ways to address this problem and compute this intervals so overcoming this limitations of probabilistic method is proposed by Arthur P. Dempster and Glenn Shafer [3]. It is now known as as Dempster-Shafer evidence theory and it caught considerable attention by AI researhes in the early 1980s, when they were trying to adapt probability theory to expert systems. The ideas on which Dempster-Shafer theory is based on are (1) the idea of obtaining degrees of belief for one question from subjective probabilities for a related question and (2) Dempster's rule for combining such degrees of belief when they are based on independent items of evidence. Both of them are considered in this paper through formalized form and examples.

However, Dempster's rule of combination has been widely criticized as it sometimes gives unreasonable results, which we showed through convenient examples. Some of the examples for modifications of Dempster's rule are given in [4] (for problems in data fusion and pattern recognition), and [6] (for an infinite number of cases where Dempster's rule does not respond adequatly even when the level of conflict is low). Here we consider modifications of Dempster's rule of combination for the problems of high conflict situations [2, 5, 8]. We pay the special to [8] which gives better results than [2, 5], as it is shown. We also offer some possibilities for it's improvement.

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SESSION 2
Complex networks

Chairman – Zoran Ognjanović

Slobodan Maletić, Milan Rajković
Simplicial Complexes as Complex Systems

Dušan Tatić, Edin Mulalić
Graph-based System for Analysis of Researchers and Paper Citations

Milan Todorović, Bojan Marinković, Aleksandar Zeljić, Paola Glavan, Zoran Ognjanović
Formal Description of the Chord Protocol using Isabelle/HOL Proof Assistant

Bojan Marinković, Vincenzo Ciancaglini, Zoran Ognjanović, Paola Glavan, Luigi Liquori, Petar Maksimović
Analyzing the Exhaustiveness of the Synapse Protocol

Aleksandar Janjić, Suzana Savić, Lazar Velimirović
Development of Energy System Smart Grids Based on Multi-Criteria Decision Making

Simplicial complexes in the research of complex systems

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Keywords

complex systems; complex networks; simplicial complexes; topological invariant; combinatorial Laplacian

Summary

Physics (and mathematics) of complex systems formed by the large number of elements interacting through pairwise interactions in highly irregular manner, is the most commonly restricted to concepts and methods of the graph theory [1]. Such systems are called complex networks and notions of "graph" and "complex network" are used interchangeably. The achievements of the complex networks research are important for modern world and largely reshape our notion of a large class of complex phenomena, primarily because seemingly random and disorganized phenomena display meaningful structure and organization. The same stands also for the aggregations of complex network's elements into communities (modules or clusters) [2], which as a major drawback has that they are restricted to the collections of pairwise interactions.

In the present research we start from typical properties of complex systems and show that simplicial complexes are the most suitable for the mathematical representation of those properties. In that way, the phenomena related with the complex systems, and hence with the complex networks, are placed in the mathematical framework of combinatorial algebraic topology. Redefining the notions of structure and substructure of complex systems, exemplified by complex networks, are given a new meaning through the changing the notion of community, by defining a simplicial community [3] which are characterized by the higher-order aggregations of network elements. It has been shown that aggregation of multidimensional simplices emerge as the natural substructure of complex network. It was further shown that simplicial complexes may be constructed from complex networks in several different ways, indicating the possible different hidden organizational patterns leading to the final structure of complex network and which are responsible for the network properties. In the present thesis two simplicial complexes obtained from complex networks are studied: the neighborhood [4] and the clique complex [3].

Topological quantities, like structure vectors, Betti numbers, combinatorial Laplacian operator are calculated for diverse models real-world networks. Properties of spectra of combinatorial Laplacian operator of simplicial complexes are explored, and the necessity of higher order spectral analysis is discussed and compared with results for ordinary graphs. The relationship of properties resulting from combinatorial Laplacian spectra with connectivity properties stored in the Q-vector is analyzed and discussed. The basic statistical features of complex networks are preserved by algebraic topological quantities of simplicial complexes, indicating possible presence of the so far unknown generic mechanisms in the complex networks formation. All results support the necessity of developing a novel research field, called statistical mechanics of simplicial complexes as a unifying theory of the complex systems represented by simplicial complexes.

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Graph-based System for Analysis of Researchers and Paper Citations

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Keywords

Citation graph; data analysis; search engine; graph visualization; citation analysis

Summary

In recent decades, there has been a rapid growth of published scientific papers. As a consequence, it became of crucial importance to have mechanisms for organizing them in an efficient way. With these tools, information can find their way and be presented differently to individual researchers. Digital libraries, such as Google Scholar, Microsoft Academic Research, DBLP etc., have made significant effort in order to enable fast search. These huge amounts of information are not only challenge for the ability to quickly retrieve data. Ranking, filtering, and recommender systems are important when dealing with large data sets. In addition, presenting results in a traditional way – as a sequence of sorted answers to a query, usually does not enable deep investigation of properties of an individual item (like time dependency, etc.). All these issues present a motivation for developing new search/results presentation standards. In order to be effective, experiments have to include data analysis customized according to specific properties of scientific literature. A particular significant topic in recent decades has been developing bibliometric measures for estimating impact of an individual paper or a journal.

In this paper, we present a system that we developed. The system enables retrieval and analysis of information about scientific literature according to these standards. Online academic database Microsoft Academic Search (MAS) is used as a data source for the developed system. Data obtained as an answer to a particular query are processed by several modules:

- Web crawler,
- Sub-graph selector,
- Data analyzer,
- Data visualizer.

Web crawler module is used for collection and interpretation of data from the Internet. For each paper, the following information is automatically collected:

- Title,
- Authors,
- Journal/Conference,
- Year of publishing,
- References.

Collected data are organized in a graph structure. Papers and authors are represented by graph nodes which are connected by edges according to citation relationships.

From the obtained graph, *Sub-graph selector* extracts specific problems as sub-graphs. The main purpose of this module is to reduce the amount of information and adapt graph structure. As a consequence, this improves performance of further data processing. The resulting sub-graph can represent citation relationships among authors, papers, or journals. In addition, it can be filtered using some parameters, for example, year of publishing.

In order to evaluate impact of an individual author or paper, various metrics have been proposed [1, 2, 3, 4]. We implemented several of these measures in *Data analyzer* module, which enables comparison among them and selection of the most appropriate for the specific application.

Visual interpretation of analyzed data is performed by the visualizer module. The visualizer is able to represent the graph structure in one of the following ways:

- Table,
- Drawn graph,
- Structure adapted for some specialized visualization tool.

Table representation is mainly for testing purposes of large-scale graphs. Smaller and middle scaled graphs are suitable for the drawn graph representation. Layout of such graphs is obtained by implementing Spring Algorithm [5, 6]. For large-scaled graph visualization, our system outputs graph in a format adequate for some of the professional tools. In this paper, Gephi software tool [7] is used for managing huge number of nodes and some further analyses.

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This is an open source software tool which supports a number of plug-ins, such as filters based on different node and edge parameters.

In this paper, we presented a system for citation analysis and discussed the technology and tools for its realization. The main idea was to retrieve data about papers and their authors from the Internet and determine influence and relations between them. Visual representation as citation graphs can help researchers to widen their view about scientific topics or papers. Scientific structure and connections among papers enable comparison of impact among various papers. Possible directions for future work include identifying critical focuses in research domains [8], clustering literature according to scientific area [9], and discovering scientific communities.

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Formal Description of the Chord Protocol using Isabelle/HOL Proof Assistant

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Keywords

Peer-to-peer; Chord; Isabelle/HOL; Formalization; DHT-based Overlay Networks;

Summary

A decentralized Peer-to-Peer system (P2P) [3] involves many peers (nodes) which execute the same software, participate in the system having equal rights and might join or leave the system continuously. In such a framework processes are dynamically distributed to peers, with no centralized control. P2P systems have no inherent bottlenecks and can potentially scale very well. Moreover, since there are no dedicated nodes critical for systems' functioning, those systems are resilient to failures, attacks, etc.

P2P systems are frequently implemented in a form of overlay networks [7], a structure that is totally independent of the underlying network that is actually connecting devices. Overlay network represents a logical look on organization of the resources. Some of the overlay networks are realized in the form of Distributed Hash Tables (DHT) that provide a lookup service similar to a hash table; (key, value) pairs are stored in a DHT, and any participating peer can efficiently retrieve the value associated with a given key. Responsibility for maintaining the mapping from keys to values is distributed among the peers, in such a way that any change in the set of participants causes a minimal amount of disruption. It allows a DHT to scale to extremely large number of peers and to handle continual node arrivals, departures, and failures. The Chord protocol [4, 5, 6] is one of the first, simplest and most popular DHTs. The paper [4] which introduces Chord has been recently awarded the SIGCOMM 2011 Test-of-Time Award.

Our aim is to verify correctness of the Chord protocol using Isabelle/HOL proof assistant. This is motivated by the obvious fact that it is difficult to reproduce errors in concurrent systems or just by program testing.

The specification of the Chord presented in this paper has been written following the implementation [1] of the high level C++-like pseudo code from [6], and the Abstract State Machine specification given in [2].

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Analyzing the Exhaustiveness of the Synapse Protocol

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Keywords

Peer-to-peer; DHT-based overlay networks; Abstract State Machines; Retrieval Probability;

Summary

Overlay networks have recently been identified as a promising model that could cope with the current issues of the Internet, such as scalability, resource discovery, failure recovery, routing efficiency, and, in particular, in the context of information retrieval. One of the main problems of overlay networking is how to allow different overlay networks to interact and co-operate with each other. When it comes to the overlay networks that have already been developed, one can perceive a great extent of heterogeneity between them. In most cases, this heterogeneity renders them unable to co-operate, communicate, and exchange resources with one another without resorting to the costly, non-scalable, and security-compromising operation that is overlay merging.

On the other hand, there are many situations where different overlay networks could benefit from co-operation for various purposes. In the context of large-scale information retrieval, several overlays may wish to offer an aggregation of their resources to their potential common users, without relinquishing control over them.

One of the possible solutions for the inter-connection of heterogeneous overlay networks is the Synapse protocol, introduced in [2, 5, 6]. It is a generic and flexible meta-protocol that provides simple mechanisms and algorithms for easy interconnection of overlay networks. The first contribution of this presentation, motivated by the ever-growing need for formal correctness, will be the formal specification of Synapse within the formalism of Abstract State Machines (ASM), introduced in [1, 3, 4] and able to simulate arbitrary algorithms (including programming languages, architectures, distributed and real-time protocols, etc.) in a direct and essentially coding-free way.

In addition to the specification of Synapse in ASM, we will provide a probabilistic estimate on the exhaustiveness of the Synapse protocol across a number of scenarios. To summarize, in this presentation we aim to: give a specification of the Synapse protocol using the formalism of ASM, theoretically analyze the exhaustiveness of the Synapse protocol, and describe and run the corresponding experiments, in order to validate the obtained theoretical results.

In doing so, we provide a starting block from which further formal analysis of the Synapse protocol can be performed, as well as an easy mechanism for estimation of the exhaustiveness of the Synapse protocol, justified by the performed experiments.

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Development of Energy System Smart Grids Based On Multi-Criteria Decision Making

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Keywords

Energy system; smart grids; energy security; multi-criteria decision making

Summary

An energy management system is a set of interconnected or interactive elements used to establish energy policy and objectives and to accomplish those objectives. Such a system is established on various levels: organization, local community, and the state. Energy management system on the level of an organization is defined by ISO 50001. The standard specifies the requirements for establishing, implementing, maintaining, and improving the energy management system, which allows the organization to continually improve energy performance and energy efficiency and to preserve energy.

The development of telecommunication and information technologies caused a rapid development of transportable and electric distribution grids. The concept of smart grids pertains to all components of the production, transfer, and distribution of electricity. Therefore, it is also necessary to define the integration of different management systems into a single complex system using the principles of interoperability aimed at [1]:

- improved functioning (connection improvement) of all system elements and technologies,
- enabling end-users to participate in system operation optimization,
- giving end-users more information on system operation and the choice of a provider,
- significantly reducing the impact of the environment on the entire electricity supply system, and
- significantly increasing the degree of supply reliability and security.

In addition, traditional distribution grids are faced with increased demands to use new technologies and to extend functionality.

A smart grid is usually defined as an electrical grid that intelligently integrates the actions of all users connected within it – producers, consumers/end-users, and those who are both, with the purpose of efficiently producing electricity and delivering it sustainably, economically, and safely.

So far, several terms have been used to denote this step forward toward new grids: advanced distribution grid, smart grid, intelligent grid, or adaptive grid.

In the EU, the concept of smart grids was adopted in 2005, as an official document of the European Commission through the European Technology Platform Smart Grids. In 2007, it was more precisely defined through the Strategic Research Agenda [2,3]. In April 2010, the Strategic Deployment Document for Europe Electricity Networks of the Future [4]. In early April 2011, the European Commission issued a statement reiterating the need to improve the existing grids, listing the following as the main objectives [5]: increased use of renewable electricity sources, grid security, energy conservation and energy efficiency, and deregulated energy market.

Unfortunately, unlike Europe, Serbia has not defined a complete strategy regarding the adaptation to new requirements and technologies.

After the pioneering project by the Electric Power Industry of Serbia "Vučje – Smart Grid City" [6], smart grid projects remained only at the level of certain, less important, aspects of remote electric meter reading. A significant step forward in this direction was made only with the projects financed by the Serbian Ministry of Education, Science and Technological Development. Only through a thus defined rigid formal framework is it possible to unite the regrettably divergent and solitary cases of smart grid development in our country.

Energy system development must be in keeping with the strategy for sustainable, competitive, and safe energy, which primarily implies: competitiveness, use of different energy sources, sustainability, innovation, and technological improvement [7]. The result of energy system development is reflected in energy performance.

Energy performance refers to quantifiable results pertaining to energy (e.g. energy efficiency, energy intensity, or specific energy consumption). Energy performance indicators are quantitative indexes of energy performance.

The key energy performance indicators were defined in 2005 as a result of cooperation between several

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international organizations – global leaders in energy and environmental statistics and analysis: International Atomic Energy Agency (IAEA), United Nations Department of Economic and Social Affairs (DESA), International Energy Agency (IEA), European Environment Agency (EEA), and the Directorate-General of the European Commission for statistics – Eurostat. The key energy performance indicators include a set of 30 indicators: 4 social indicators, 16 economic indicators, and 10 environmental indicators.
The American Energy Institute combines 37 metrics within four sub-indexes that define the four major areas of energy security: geopolitical, economic, reliability, and environmental. These are subsequently used to determine the aggregate index of energy security and are weighted as follows: geopolitical 30%, economic 30%, reliability 20%, and environmental 20%. The values of the U.S. Energy Security Risk Index were determined based on the data for the period between 1970 and 2010, and predicted for the period between 2011 and 2035 [8].
The indicator values do not merely represent data but the basis for communication between stakeholders regarding sustainable energy use. Each set of indicators (social, economic, or environmental) expresses specific aspects or impacts of energy production and use.
From the standpoint of manufacturers of intelligent power grid, the most important standards are those dealing with the technical details of information security applicable to the design of the device. To IEC 62 443 are in the field of SCADA devices, IEC 62 351 in the field of intelligent reading, and NIST 800-53 in validation, testing and documentation of computer security.
On the other hand, for the comprehensive information security policy standardiziovanje elektrodistributivnog most companies are ISO/IEC 27000, CPNI recommendations, and the NERC CIP standards. For electric power companies in Europe the selection of IT security standards is clear - ISO / IEC 27000. It must be the basis for risk assessment, security policies and plans to control and overcome the risks. In practice, the problem arises with the implementation of this standard because it is too generic. It is therefore recommended to use other standards, particularly NERC CIP and CPNI recommendations where necessary to develop specific procedures and plans to overcome the risks.
The choice of adequate activity for planning smart grid development is a complex and difficult task. This is due to three main reasons: (1) presence of various alternatives; (2) existence of multiple criteria (economic, technical, environmental, etc.) to be met simultaneously, although they are often incommensurable and incomparable; (3) renewable energy sources in a distribution grid must be optimized based on operational needs and on algorithms used for the optimization.
Consequently, it is necessary to perform a multi-criteria analysis in the process of smart grid development [11].
This paper proposes a new algorithm, which uses the fuzzy max-min and AHP methods for multi-criteria decision making. Based on dynamic fuzzy matching of alternatives, the method determines the activity timetable for distribution system planning.
The method is illustrated on the example of equipment replacement in 35/10 kV distribution substations.
We proved that the method is highly successful in the evaluation of alternatives in the presence of heterogeneous criteria.

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SESSION 3
Complex systems and stochastic processes

Chairman – Miomir Stanković

Ivan Živković, Predrag Stanimirović
Modelling Neural Networks as Dynamical Systems for Matrix Inverse Computation

Nataša Glišović
Predicting Cash Flows by Using Neural Networks

Nataša Glišović
Optimization Problems Time/Cost Tradeoff by Using Monte Carlo Methods

Predrag Popović
Option Pricing Using Monte Carlo Maximum Entropy Approach

Suzana Savić, Miomir Stanković , Goran Janaćković
Fuzzy AHP Ranking of Occupational Safety System Quality Indicators

Modelling Neural Networks as Dynamical Systems for Matrix Inverse Computation

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Keywords

dynamical system, artificial neural network, control system, parallel distributed processing, generalized inverse.

Summary

Lately, neural networks have shown huge potential as parallel distributed computational models for solving many computationally challenging problems. It is demonstrated that neural networks can be used effectively for the identification and control of nonlinear dynamical systems. Approach for identification of nonlinear dynamic system using neural networks is to involve the dynamic differential equation into each of the neural network processing elements to create a new type of neuron called a dynamic neuron. Since differential equations are involved in the processing, these approaches cannot take full advantage of the neural network operation. For structural dynamic model identification, the knowledge of system dynamics is useful. Each neuron i is characterized by its state, X_i, which belongs to some compact set $I \in \mathbb{R}^M$. M is the number of variables characterizing the state of one neuron(we assume that all neurons are described by the same number of variables). A typical example is $M = 1$ and $X_i = V_i$ is the membrane potential of neuron i and $I = [V_{min}, V_{max}]$.
We consider the evolution of N neurons, given by a deterministic dynamical system of type

$$\frac{dX(t)}{dt} = \mathbf{F}_\gamma(X, t)$$

The variable $X = \{X_i\}_{i=1}^N$ represents the dynamical state of a network of N neurons at time t. We use the notation V instead of X when neurons state is only determined by membrane potential whereas we use the general notation X when additional variables are involved.

Recently, a number results related to the application of neural networks in solving a variety of matrix algebra problems has been published. Different types of neural networks have been introduced to solve systems of linear algebraic equations. The authors of the papers [3, 15] proposed a recurrent neural networks for solving simultaneous linear algebraic equations. Wang in [12, 13, 16] proposed a gradient neural network to solve simultaneous linear equations. In [16], it is verified that proposed recurrent neural networks are asymptotically stable in the large and capable of computing inverse matrices and solving Lyapunov matrix equations. Two three-dimensional structured networks for solving linear equations and the Lyapunov equation are developed in [19].

Neuron-like network architectures for computing eigenvalues and eigenvectors of real matrices is investigated in [4, 10]. Two recurrent neural networks for computing LU decomposition and Cholesky factorization are presented in [17]. Also, in [17] it is proven that the proposed recurrent neural networks are asymptotically stable in the large.

A variety of other matrix algebra problems has been solved by using neural networks in [1, 5, 20].
Matrix inversion has been widely used in a variety of applications such as robotics, control, and signal .processing. For many large-scale problems, the orders of the matrices are very large and large-scale inverse matrices often need to be computed in real-time for monitoring and controlling dynamic systems. Large matrix inversion using existing algorithms in real-time is usually not efficient due to the nature of the sequential processing. For such applications, parallel distributed processing is more desirable.

Also, nonlinear and linear recurrent neural network models have been developed for the inversion and generalized inversion of square and full-rank rectangular matrices in [7, 8, 14, 16]. A number of various recurrent neural networks for computing generalized inverses of rank-deficient matrices are developed in [18, 21].

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The Moore–Penrose inverse is a generalization of the usual inverse of a nonsingular matrix to the set of singular or rectangular matrices. The Drazin inverse is a kind of generalized inverse of a matrix only defined for square matrices. This kind of inverse has been applied to various fields, for instance, singular linear systems, finite Markov chains, singular differential and difference equations, multibody system dynamics etc. (see, [2, 6, 9, 11]). In the monograph [2] as well as in the papers [22, 23] it is showed that the Drazin inverse solution solves a linear system $Ax = b$ under certain conditions. It is also known result that the Drazin inverse solution represents the minimal P-norm solution of the linear system $Ax = b$ [22].

In this paper, we develop a linear recurrent neural network for computation of the generalized inverse of a singular matrix as well as for computation of the inverse of a regular matrix. Similarly with the recurrent neural networks for matrix inversion developed earlier, proposed neural network is also composed from a number of independent sub-networks corresponding to columns of a pseudo inverse matrix. The proposed recurrent neural network is shown to be capable of computing the generalized inverse of a singular matrix. The performance of the proposed recurrent neural networks is demonstrated by means of several numerical examples.

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Predicting Cash Flows by Using Neural Networks

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Keywords

Neural networks, back propagation, cash flows, prediction.

Summary

The report about the cash flow is an important thing of the analysis considering the fact that the possibility of manipulation with accounting rules, as far as incomes and expenses are concerned, is significant which reduced the efficiency of the success balance as the analysis subject and increased the influence of the reports about the money flows. This report is regulated MRS 7. The report about the money flows and makes the obligatory financial report. Cash flows are inflows and outflows of cash and cash equivalents. Inflows and outflows of cash arising through, checking accounts (local currency and foreign currency), including of compensation, assignment and assignment carried out through these accounts. Neural networks can be used for prediction, classification, and association in various problem areas. We chose to test them on a projection of cash flows because of the presence of non-linearity and uncertainty. Reason for choosing neural networks is that they are robust tool designed to work with large amounts of fluctuating data and data with disabilities, including hidden non-linear dependence. Neural networks are composed of two or more layers or a set of processing elements called neurons. Neurons are connected to the network in such a way that the output of each neuron is input to one or more other neurons. There are three main types of neuron layers: input, hidden and output. The input layer receives input from "outside" of the environment and sends it to one or more hidden layers. In the hidden layer neurons of the information is processed and sent to the neurons in the output layer. Information then travels back through the network, and the values of weight connections between neurons are adjusted according to the desired output. The process is repeated on the network in as many iterations as necessary to reach the nearest exit is desired (real) output. Neural network is basically a process in which the system comes to the value of weight connections between neurons. The aim of this research is the survey of the system for the projection of cash flows using neural networks. The research results are applied to example of a real firm. The idea of the research is to show the ability of predicting of "moving" AOP positions by using Neural network back propagation.

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Optimization problems time/cost tradeoff by using Monte Carlo methods

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Keywords

Project management, Time-cost trade-off problem, Fuzzy-Monte Carlo algorithm.

Summary

The goal of project is to shorten the project duration and cost. Uncertainties are always a reason to delay the project and exceeding budget. Difficult to balance the duration of the project and its cost. Knowing the critical path, project manager and his team can use a variety of techniques to shorten the duration of the project. Crashing is a technique that allows an optimal reduction of time and cost to the project. It reduces the time with the least expense. This technique is also known as time cost tradeoffs. Crashing is the process of reducing the duration of critical path activities by allocating more resources to those activities or change their size. Some activities (or all) can be accelerated by increasing the funds. The aim is to reduce the cost and time of the project. In project management, time and cost are two control factors. The project should be completed on time. However, the project implementation is influenced by the uncertainty that extends the life of the project. Delay produces a number of effects on the poor performers that cause loss of profits. The problem of optimization of this problem is NP hard. Therefore, most scholars resolve this problem heuristic methods. The aim of this paper is to solve this problem by the fuzzy Monte Carlo method. The results are applied to a real example. One of the most popular applications of the Monte Carlo algorithms is in the field of finance. Monte Carlo methods aid the analysis of financial instruments, portfolios, and assets. When a financial instrument or asset is being analyzed to label its value, many complex equations, the values of which may be uncertain, are used to reach a final answer. Since Monte Carlo methods work well with highly complex equations, their use becomes vital in the calculation of uncertain values, which then in turn help analyze the final value of the instrument or asset in question. A specific 'Monte Carlo Option Model' is used to evaluate future prices of options. Many uncertain values affect the final value of these financial options; Monte Carlo methods use random number generation to lay the various price paths and then calculate a final option value. The study proposes a framework incorporating the Fuzzy Monte Carlo simulation approach to analyze the project completion probabilities among various time/cost constraints.

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Option Pricing Using Monte Carlo Maximum Entropy Approach

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Keywords

option pricing, maximum entropy distribution, monte carlo

Summary

In the world of finance, pricing options arises as an every day issue. Since there is a great variety of types of options investors face with problems that can be very complex. One of the successful technic in option pricing is Monte Carlo method. By simulating the underling asset's price we can predict the price of the option at any time. Difficulties that arise here is in simulating the evolution of the price. If we can capture behavior of market data with mathematical model we will be able to understand what is going to happened after some time period. These kind of models are probabilistic. Tendency to model these data with normal distribution are often misleading. Because of skewness and leptokurtic behavior some alternative approaches are needed. Thus, we suggest a density functions based on maximum entropy approach. We discus q-Gaussian density function and suggest general density function derived from maximizing Shannon's entropy after introducing additional parameter. Some discussion on these functions can be found in [6] as well as application on the real data from the NYSE. Practical aspect of these functions are also discussed in [2] and [5].

In this paper we focus on pricing American options. American option is a contract to buy or sell an underlying asset under predefined price at any time during the contract validity. Since the options can be executed at any time the problem of pricing the security becomes very complex. Future payoff depends not only on final value of the underlier but on all values it takes during the life of the contract. One successful method to price American options is presented in [3]. This method is based on the least square Monte Carlo approach but it is derived under the assumption that underlying assets evolve under normal distribution. Further properties of this algorithm is discussed in [4]. We implement this method based on maximum entropy distribution functions. By using q-Gaussian function we are able to model non gaussian properties by changing value of the parameter q. On a simple model we explain how this algorithm works. Further, we perform simulation of this algorithm on a real option pricing problem and discuss obtained results. Some aspects of evolution of stock prices under maximum entropy distribution is also presented in [1]. We also suggest further modifications of the method. By introducing additional parameter we extend simple exponential solution forms that figures in maximum entropy distribution to more general exponential forms that are more flexible and can be adjusted to model a bigger scope of a data.

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Fuzzy AHP Ranking of Occupational Safety System Quality Indicators

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Keywords

Occupational safety system; quality indicators; fuzzy analytic hierarchy process (fuzzy AHP)

Summary

A complex system is any system that contains a large number of interactive elements (agents, processes, etc.) that have a non-linear resulting behavior and usually has a hierarchical self-organization [1, 2]. The main problem is the problem of complex system modeling and simulation. Therefore, during the research of these systems are often used multi-criteria analysis methods based on expert judgments.
Occupational safety system consists of various elements: employees (with different individual and group behavior, habits, and culture), the object of labor (material, energy, and information), means of production (machinery, equipment, tools) and work environment (in which forming material and social working conditions). By interactions among the elements at various levels are formed different system behaviors, often non-linear. Therefore, safety system is a complex system. The quality of the system is defined by performance and indicators as its measures [3, 4].
The paper describes the safety system as a complex system. We present factors, performance and indicators of occupational safety system, as well as methods for selection and ranking of occupational safety indicators - expert evaluation method and fuzzy analytic hierarchy process. The case study was performed in road companies in Serbia.
Indicators of occupational health and safety have been investigated by many authors [5-11]. Based on these studies and on the basis of their research [3, 4], a list of 48 indicators is created. The indicators are divided into four groups: technical (11 indicators), human (15 indicators), organizational (16 indicators) and environmental (6 indicators). Key performance indicators are selected on the basis of assessment of experts from the Institute of the quality of living and working environment "1. May" from Nis. They assessed risks in many road companies in Serbia. After the selection of key indicators by means of expert evaluation (selected 20 of the proposed 48 indicators), the problem is solved by their ranking with fuzzy AHP method.
The original AHP method developed T. Satty [12]. It is a method of multi-criteria decision-making that includes both objective and subjective factors, and in which the expert evaluation has real numbers. However, in many practical cases, the preferred model is uncertain and decision makers cannot use real numbers to express preferences. Thus, there was an expansion of the basic model – the fuzzy AHP method [13]. This method allows the use of incomplete information and presenting them in the form of fuzzy numbers. Using fuzzy numbers to evaluate the occupational safety system quality indicators allows more realistic view of the problem [14].
In this paper, fuzzy AHP method was applied by means of the following steps: 1 Goal definition; 2 Identification of criteria, sub-criteria and alternatives; 3 The formation of hierarchical structures; 4 Pairwise comparison using fuzzyfied Satty's evaluation scale; 5 Determination of priority vectors using the eigenvalue method, fuzzy analysis and the principles of aggregation; 6 Defuzzyfication with total integral value method and the final ranking of alternatives.
The problem is represented by a hierarchical structure with four levels: the first level is the task (ranking of key indicators); the second level are relevant criteria (risk, cost, social responsibility), the third level are the sub-criteria (technical, human, organizational and external factors); and the fourth level are the key indicators (20 in total).
According to the analysis of the results of pair-wise comparisons of criteria, sub-criteria with respect to each criterion, and indicators within the relevant sub-criteria, the corresponding priority vectors are formed. After the defuzzyfication of the obtained values, the following can be concluded:

1. Safety at work is primarily based on the estimated risk. The other two criteria, cost and social responsibility are much less significant.
2. In relation to the risk, human and organizational factors are dominant, followed by technical factors and environmental factors. Compared to the cost, the most important are technical factors, then organizational, external, and, finally, human factors. Organizational and external factors are dominant in relation to social responsibility, followed by human and technical factors.
3. Among the technical indicators, the most important indicator is the number of safety levels, among the

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human indicators - the level of compliance with operating procedures, among the organizational indicators - the efficiency of safety resources, and among the external indicators - the level of safety technologies.

The resulting priorities of the sub-criteria are: organizational factors, external factors, human factors, and technical factors.
The following resulting rank order of the most important indicators is obtained: the efficiency of safety resource management; the number of safety controls in practice; the level of safety technologies; the level of compliance with operating procedures; and the proportion of jobs at risk.
Definition of key indicators of occupational safety system, their ranking by suitable methods and continuously monitoring and improvement of the value of the best ranked indicators improves the quality of occupational safety system, and thus the competitiveness of the organization [15, 16].

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SESSION 4
Signal processing

Chairman – Velimir Ilić

Dragorad Milovanović, Zoran Bojković
Operational Rate-Distortion Information Theory in Optimization of Advanced Digital Video Codec

Zoran Perić, Jelena Nikolić , Vladica Djordjević
Linearization of the Optimal Compressor Function for Gaussian Mixture Model

Lazar Velimirović, Zoran Perić, Bojan Denić
Design and Analysis of the Two-level Scalar Quantizer with Extended Huffman Coding

Operational Rate-Distortion information theory in optimization of advanced digital video codec

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Keywords

coding efficiency; operational configuration; codec complexity; video quality

Summary

The relevant results of information coding theory, operational *Rate-Distortion* framework and optimization techniques in research and development of advanced digital video codec, are reconsidered in this paper. In the first part, we present an evolution of methods based on theory of coding with information distortion and formulate optimization problems in an efficient coding of digital video signals. We selected foundamental references in mathematical theory of telecommunications, deterministic dynamic programming and discrete versions of the *Lagrange* multiplier method, as well as their original applications in coding signals. In the second part, we study in-depth MPEG operational framework of standardized hybrid digital video codec. The *quantization parameter* QP is optimized under constraints of maximum available bitrate *R*, in a way that minimize MSE distortion *D* of decoded video. The operational parameters of codec are adaptively selected from finite set of possible video frame partitions for prediction, transform coding and available quantizers.

Classical coding theory with information distortion predicted the existence of the lower asymptotic limit of average signal distortion (*D*) signals under constrained average bitrate (*R*). Practical trade-off between the allowed distortion *D* and available bitrate *R* in designing an encoder, is based on the *optimization procedure* of finding a local optimum operational (R, D) points. Standard based encoder requires *discrete* optimization procedure over a set of allowed *operating parameters* as well as additional optimization criteria that arise from real-time operations (complexity, delay). The goal of operational information theory is to find a set of operating parameters of the encoder which is optimal in *R*(*D*) sense, as well as efficient optimization procedure based on a fast algorithms solution of the full search of parameter's space [1, 2, 3, 4, 5, 6].

We applied an unified approach to the operational analysis in computer simulation of three generations of MPEG/ITU digital video coding encoders [7]. The coding efficiency of HEVC/H.265, AVC/H.264, and H.263 reference software implementation is compared by means of PSNR distortion measure and subjective quality in Internet applications (video chat, video conferencing, telepresence systems). Bitrate reduction BRr and coding gain CG based on PSNR measure and complexity based on encoding/decoding time of HEVC MP vs. AVC HP vs. H.263 CHC are tested at bitrates of 0.256, 0.384, 0.512, 0.850, and 1.500 Mbps using the low-delay operational constraints typical for real-time conversational applications. Low-delay coding are considered by selecting operational prediction structures and operational options in software configuration of encoders. Real-time decoding complexity where studied on personal *Ultrabook* x86-64 computer with standard set of test video sequences. The results of performance tests for selected format HD720 video sequences indicate that third generation HEVC encoders achieve equivalent objective *D* video quality as encoders that conform to AVC when using approximately 60% less bitrate *R* on average [8].

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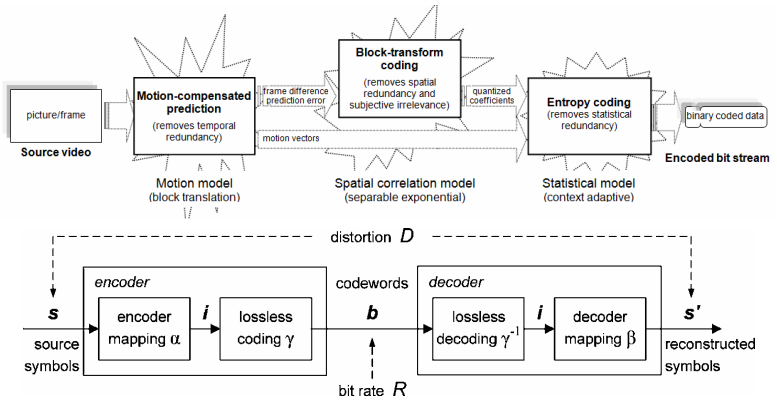


Fig. 1 Digital video source coding: perceptual hybrid encoder and operational R-D model [1,2].

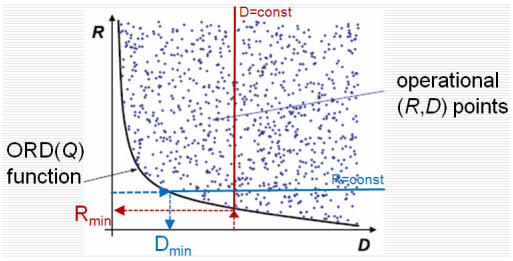
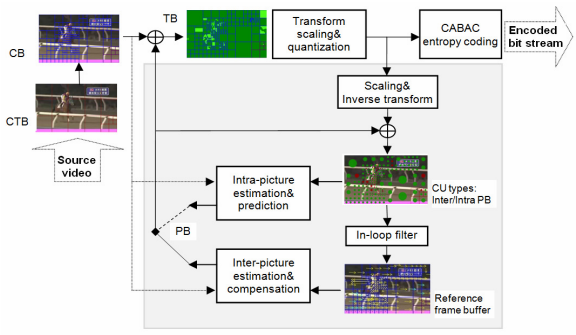


Fig. 2 Operational R-D function is a convex border of available (*R*,*D*) working points of codec [7].



Quantization Parameter QP Bitrate [Mbps] PSNR [dB]	HEVC	AVC	H.263
QP_1	20	20	2
QP_2	22	22	3
QP_3	24	24	4
BR	1.292	1.709	
$PSNR_Y$	42.49	42.04	
$PSNR_U$	46.17	46.00	
$PSNR_V$	47.02	46.81	

R(QP)
D(QP)

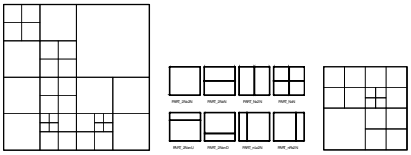
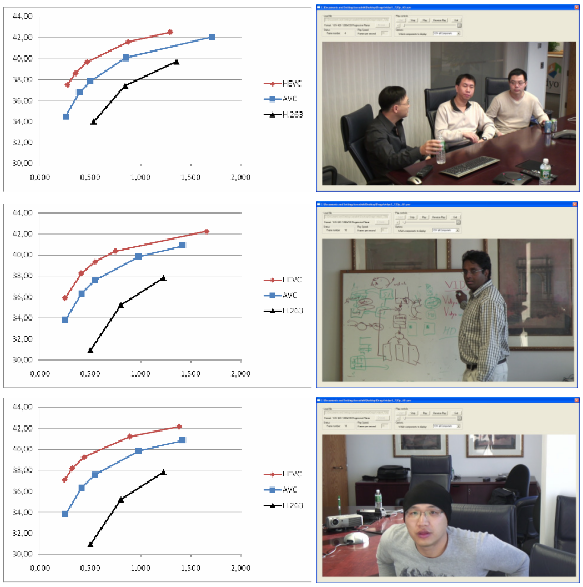


Fig. 3 Standardized MPEG-H HEVC / ITU H.265 video codec: Block-based hybrid transform-entropy video encoder with motion-compensated prediction; Operational ORD points for 12 different quantization parameters *QP* and bitrates *R* in encoding/decoding test sequence *Vidyo1;* Adaptive frame partition into Coding Blocks (CB), PredictionBlocks (PB), TransformBlocks (TB) [8].



HD720 *Vidyo1*	HEVC	PSNR=41.25dB BR=0.512Mbps	PSNR=42.59dB BR=0.850Mbps	PSNR=43.79dB BR=1.500Mbps
	AVC	BRr=0.329 64.26%	BRr=0.508 59.76%	BRr=0.910 60.73%
	H.263	BRr=0.928 181.25%	BRr=1.275 150%	BRr=1.895 126.33%
HD720 *Vidyo2*	HEVC	PSNR=40.65dB BR=0.512Mbps	PSNR=42.04dB BR=0.850Mbps	PSNR=43.29dB BR=1.500Mbps
	AVC	BRr=0.289 56.44%	BRr=0.478 56.24%	BRr=0.860 57.33%
	H.263	BRr=1.142 223.00%	BRr=1.658 200.00%	BRr=2.318 154.53%
HD720 *Vidyo3*	HEVC	PSNR=41.19dB BR=0.512Mbps	PSNR=42.42dB BR=0.850Mbps	PSNR=43.60dB BR=1.500Mbps
	AVC	BRr=0.316 61.72%	BRr=0.495 58.24%	BRr=0.900 60%
	H.263	BRr=1.054 205.86%	BRr=1.398 164.47%	BRr=2.016 134.40%

Fig. 4 Simulation results in performance comparison of HEVC MP@LD vs. AVC HP vs. H.263 CHC of the HD720 *Vidyo1/2/3* test sequences encoding: Operational Rate(QP)-Distortion(QP) functions $PSNR_Y$(BR) and HEVC subjective quality gain encoded at bitrate BR~0.512Mbps; Coding gain BitRate reduction [Mbps] based on ORD function interpolation and PSNR=*const*.

Linearization of the Optimal Compressor Function for Gaussian Mixture Model

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Keywords

Gaussian mixture model; Linearization; Non-uniform quantizer; Optimal compressor function; Scalar quantization

Summary

Scalar quantization is a process of a discretization of the current value of the continuous signal along the amplitude continuum and it must precede the coding [1-6]. The device that is used for scalar quantization is called a scalar quantizer. Two basic types of scalar quantizers are uniform and non-uniform quantizers [4]. A uniform quantizer is characterized by equal spacing between discrete amplitude levels and it is used in the case when the input signal has a uniform probability density function. A non-uniform quantizer is characterized by unequal spacing between discrete amplitude levels and it is used in the case when the input signal has a non-uniform probability density function (e.g. speech signal). To reduce the amplitude dynamic of signal, the process called compression is used. Compression is a process that is equivalent to a non-uniform quantization, because the non-uniform quantization can be achieved by uniform quantization of the previously compressed current values of the input signal [1,4]. The most commonly used compressor functions are optimal compressor function, quasi-logarithmic μ-law and semi-logarithmic A-law compressor functions [4]. The advantage of optimal compressor function is the maximum value of the SQNR (signal-to-quantization-noise ratio), and the advantage of logarithmic compressor functions, based on μ-law and A-law, is constant value of the SQNR in the wide range of the input signal variance. On the receiving side of system, it is necessary to restore the natural ratio of the amplitude values of the signal, and for this reason, the device called expander, which is characterized by the characteristic which is inverse to the compressor characteristic, is used. Because of the very complex practical realization of compressor function, and thus its inverse function, its linearization must be performed. Thus the quantizer range is divided into segments and within each segment the compressor function is approximated by a linear function [7,8]. In this paper, the linearization of the optimal compressor function was done by using a method with a variable number of representational levels within segments. The significance of this work lies in the fact that in order to achieve a better approximation of the histogram of probability density function of real signals, than in the case of Laplacian and Gaussian sources [7,8], the Gaussian mixture model, which consists of two Gaussian components, was used [4]. This is reflected in the significantly higher value of SQNR than in the case of linearization of the optimal compressor functions reported in [7,8].

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Design and analysis of the two-level scalar quantizer with extended Huffman coding

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Keywords

Entropy coding; Huffman coding; Quantization

Summary

Entropy coding is a type of lossless coding to compress digital data by representing frequently occurring patterns with few bits and rarely occurring patterns with many bits. Two most popular entropy coding schemes are Huffman coding and arithmetic coding [1]. The basic idea in Huffman coding is to assign short codewords to those input blocks with high probabilities and long codewords to those with low probabilities. Extended Huffman coding is the procedure of determining the optimal length of codewords for blocks of two or more symbols. In this paper we concerned with blocks of two, three, four and five symbols [1].

In this paper we propose a model of the two-level scalar quantizer with extended Huffman coding and variable decision threshold. We decide that the new quantizer model has only two representation levels due to small model complexity and the possibility of the efficient use of the Huffman coding procedure. Variable decision threshold is proposed so the representation levels' assymetry can be achieved. The basic idea described in this paper is that, unlike to the Lloyd-Max's quantizer, the asymmetry of the representation levels is assumed such that to provide an unequal probability of representation levels for the symmetric Gaussian probability density function (PDF) [2], [3], [4]. Representation levels are determined from the centroid condition. Variable decision threshold is determined depending on signal quality that wants to be achieved. The proposed quantizer model is optimal when the variable decision threshold is equal to zero [1], [2]. The goal of designing the proposed model is the approaching of the average bit rate to the source entropy as close as possible.

The performances of the quantizer are often determined by SQNR [1], [2]. The optimal SQNR value of the Lloyd-Max's quantizer having two quantization levels is 3 dB for Laplacian PDF, and 4.3965 dB for Gaussian PDF [2]. Therefore, the SQNR range in which we consider the performance of the proposed quantizer is from 3 dB to 4.3965 dB, that is we considered the range of decision variable t_1 from 0 to 1.2 with step 0.1.

Comparing numerical results for the proposed quantizer and results obtained in case when the signal at the entrance of the proposed quantizer is described by Laplacian PDF, it is shown that better performances are achieved with the proposed quantizer model for Gaussian PDF [5]. Also, it is shown that with the increase of symbol blocks' number that make one block the average bit rate of the proposed quantizer with extended Huffman coding converges more closely to the source entropy. However with the symbol increase in one block, the complexity of designing the proposed quatizer model increases too. Therefore it is important to compromise between the complexity of designing the proposed quantizer model and signal quality that wants to be achieved.

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