

The Second Conference on Information Theory and Complex Systems  
TINKOS 2014

# BOOK OF ABSTRACTS

Editors: Velimir Ilić and Miomir Stanković



Niš, Serbia, June 16-17, 2014

Mathematical Institute of the Serbian Academy of Sciences and Arts

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Република Србија  
МИНИСТАРСТВО ПРОСВЕТЕ  
НАУКЕ И ТЕХНОЛОШКОГ  
РАЗВОЈА

The conference is organized by  
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# THEMATIC FIELDS

- Information and complexity measures. Physical aspects of the information theory.
- Classical, non-extensive and quantum information theory.
- Source and channel coding.
- Cryptography and data security.
- Complex systems. Dynamic systems.
- Self-organization. Causality in complex systems.
- Complex networks.
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# Probability Logics

Zoran Marković<sup>1</sup>, Zoran Ognjanović<sup>2</sup>, Miodrag Rašković<sup>3</sup>

<sup>1,2,3</sup>*Matematički institut SANU, Kneza Mihaila 36, Belgrade, Serbia*

*E-mail:* <sup>1</sup>zoranm@mi.sanu.ac.rs, <sup>2</sup>zorano@mi.sanu.ac.rs

## Abstract

Axiomatizations, completeness, compactness and decidability of a number of logics, proposed by the authors, are addressed. Some possible applications are analyzed.

**Key words:** conditional probability, approximate probability, strong completeness, decidability, default reasoning

## Synopsis

The problem of reasoning with uncertain knowledge is an ancient problem dating, at least, from Leibnitz and Boole. However, in the last decades there is a growing interest in the field connected with applications to computer science and artificial intelligence. Researchers from those areas have studied uncertain reasoning using different tools. Some of the proposed formalisms for representing, and reasoning with, uncertain knowledge are based on probabilistic logics. That approach extends the classical (propositional or first order) calculus with expressions that speak about probability, while formulas remain true or false. The probability operators behave like modal operators and the corresponding semantics consists in special types of Kripke models (possible worlds) with addition of probability measures defined over the worlds. One of the main proof-theoretical problems with that approach is providing an axiom system which would be strongly complete ("every consistent set of formulas has a model", in contrast to the weak completeness "every consistent formula has a model"). This results from the inherent non-compactness of such systems. Namely, in such languages it is possible to define an inconsistent infinite set of formulas, every finite subset of which is consistent (e.g.,  $\{\neg P_{=0}\alpha\} \cup \{P_{<1/n}\alpha : n \text{ is a positive integer}\}$ ). There is an unpleasant consequence of finitary axiomatization in that case: there exist unsatisfiable sets of formulas that are consistent with respect to the assumed finite axiomatic system (since all finite subsets are consistent and deductions are finite sequences). Another important theoretical problem is related to the decidability issue.

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# Random Ergodic Operators and Information Theory

Venceslav Kafedziski

*Faculty of Electrical Engineering and Information Technologies, University Ss Cyril and Methodius, Skopje, Republic of Macedonia*

*E-mail: kafedzi@feit.ukim.edu.mk*

## Abstract

Since the original Shannon work on channel capacity and rate distortion, large number of channel and source models have been constructed, for which channel coding limits (capacity) and source coding limits (rate distortion) have been derived. When evaluating asymptotic performance limits for infinite blocks, both problems boil down to evaluation of limiting eigenvalue distribution functions (LEDF) of matrices of increasing size. We consider Single Input Single Output (SISO) and Multiple Input Multiple Output (MIMO) finite impulse response (FIR) channels and scalar and vector autoregressive (AR) sources, where FIR channel coefficients or AR source parameters vary according to stationary, ergodic vector processes. We prove that the LEDF exist in the case of SISO and MIMO channel matrices and scalar and vector AR source matrices using the theory of random ergodic operators, widely used in physics to study disordered systems. The capacity of SISO and MIMO channels and the rate distortion of scalar and vector AR sources are evaluated as parametric Stieltjes integral relations with respect to the LEDF.

**Key words:** Capacity, Rate Distortion, Limiting Eigenvalue Distribution Function, Random Ergodic Operators

## Synopsis

Capacity evaluation for fixed FIR channels uses eigendecomposition of the channel matrices to obtain a vector Gaussian channel [1]. For rate distortion of AR sources, the matrix of the AR parameters is used [2]. All these matrices are (asymptotically) Toeplitz, for which LEDFs are known and given by the celebrated Szego theorem.

Here, we address the capacity of SISO and MIMO FIR channels and the rate distortion of scalar and vector AR sources, where all the coefficients vary according to stationary, ergodic processes. In these cases the matrices are not (asymptotically) Toeplitz. Using the theory of random ergodic operators, we prove that the LEDFs, as the weak limits almost everywhere on the sample space where the channel or source random processes are defined, for all the mentioned cases exist. The existence of LEDF implies the existence of the capacity and rate distortion [3, 4].

We now define a random operator and random ergodic operator on a Hilbert space [5].

**Definition 1:** Let  $D$  be a dense linear subspace of the Hilbert space  $\mathcal{H}$ . A random operator with domain  $D$  is a map  $A$  from the set of realizations  $\Omega$  into the set of linear operators on  $\mathcal{H}$ , such that  $D$  lies in the domain of  $A$  with probability 1, and  $Ax$  is a random vector in  $\mathcal{H}$  for all  $x \in D$ .

**Definition 2:** A random operator  $A$  on a Hilbert space  $\mathcal{H}$  is called ergodic if there exists a homomorphism from a metrically transitive group  $\mathcal{T}$  of automorphisms of the probability space  $(\Omega, \mathcal{F}, P)$  into a group  $\mathcal{U} = \{U_T : T \in \mathcal{T}\}$  of unitary operators on  $\mathcal{H}$ , such that  $A(T\omega) = U_T A(\omega) U_T^{-1}$ .

All the problems that we discuss involve description in terms of random ergodic operators with finite number of diagonals (Jacobi random ergodic operator), obtained by multiplying the operator describing the channel or the AR source with its adjoint. We define a sequence of matrices  $A_N = \{a_{i,j}\}$ ,  $|i|, |j| \leq N$  of size  $(2N + 1) \times (2N + 1)$ ,  $N \in \mathbb{Z}_+$  obtained by truncating the operator  $A = \{a_{i,j}\}$ ,  $i, j \in \mathbb{Z}$  around the origine. Define the empirical distribution function  $F_N(\lambda)$  of the eigenvalues  $\lambda_{N,i}$  of  $A_N$  as

$$F_N(\lambda) = \frac{1}{2N + 1} (\text{number of } \lambda_{N,i} \leq \lambda)$$

Define random matrix norm  $\|A\|_{1,1} = \sum_{i \in \mathbb{Z}} E[|a_{i,0}|]$ . By the Theorem 4.8 in [5],  $\|A\|_{1,1} < \infty$  implies the existence of the LEDF  $F = \lim_{N \rightarrow \infty} F_N$  (weak convergence a.e. on the process sample space). The operator is approximated by a sequence of bounded operators. Convergence in  $\|A\|_{1,1}$  norm implies convergence of Stieltjes transforms of the operator spectral measures, which implies convergence of spectral measures and of distribution functions. Jacobi matrices with  $E[|a_{i,0}|] < \infty$  satisfy  $\|A\|_{1,1} < \infty$ . In all the considered cases this condition is satisfied.

**Theorem 1:** Capacity  $C$  in terms of power  $P$  of SISO FIR channel with ergodic coefficients is given by the parametric Stieltjes integral relations:

$$C = \int_0^\infty \log[\max(\theta \lambda \sigma_w^{-2}, 1)] dF(\lambda)$$

$$P = \int_0^\infty \max(\theta - \lambda^{-1} \sigma_w^2, 0) dF(\lambda)$$

**Theorem 2:** Rate distortion  $R(D)$  of a scalar AR source is given by the parametric Stieltjes integral relations:

$$D = \int_0^\infty \min\left(\frac{1}{\beta}, \theta\right) dF(\beta)$$

$$R = \frac{1}{2} \int_0^\infty \log \max\left(\beta, \frac{1}{\theta}\right) dF(\beta)$$

Besides the existence of the LEDFs, in order to prove the existence of capacity and rate distortion, it is necessary to show that  $\int_0^\infty f dF_N \rightarrow \int_0^\infty f dF$  for all the functions  $f$  involved. In fact, all the functions involved are bounded or increase at a rate less than linear, and, thus, meet the convergence condition [6].

Both theorems extend to vector cases (MIMO, vector AR source) by introducing block stationary processes on the diagonals of the operator. All the proofs are modified such that the process measure is substituted by its process stationary mean [7]. In the condition  $E[|a_{i,0}|] < \infty$ , the expectation is now with respect to the process stationary mean and is represented as a finite sum of expectations with respect to the original process measure, and, thus, the condition is still met. The equations in both theorems stay the same, only the LEDFs change.

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# LDPC Codes and Message-Passing Decoders: An Introductory Survey

**Valentin Savin**

*CEA-LETI, MINATEC Campus, 17 rue des Martyrs, 38054 Grenoble, France*

*E-mail: valentin.savin@cea.fr*

## Abstract

The outstanding success of Low Density Parity Check (LDPC) codes in providing practical constructions that closely approach the theoretical Shannon limit is rooted in the way they are decoded. They feature iterative message-passing decoders able to convey information between coded bits, so that to progressively improve the estimation of the sent codeword. This tutorial provides first an overall survey of LDPC decoders, and then a more detailed insight into some of the most widely used decoders. We also discuss the asymptotic analysis of these decoders and explain how this analysis made possible the optimization of LDPC codes operating very close to the Shannon limit.

**Key words:** LDPC codes, Iterative decoders, Message-Passing, Belief-Propagation, Sum-Product, Min-Sum.

## Synopsis

It is widely recognized that one of the most significant contributions to coding theory is the invention of Low-Density Parity-Check (LDPC) codes by Gallager in the early 60's [1]. Yet, rather than a family of codes, Gallager invented a new method of decoding linear codes, by using iterative message-passing (MP) algorithms. Such a decoding algorithm consists of an exchange of messages between coded bits and parity checks they participate in. Each message provides an estimation of either the sender or the recipient coded bit, and the exchange of messages takes place in several rounds, or iterations. At each iteration, new messages are computed in an *extrinsic manner*, meaning that the message received by a coded bit from a parity-check (or vice versa) does not depend on the message just sent the other way around. Consequently, coded bits collect more and more information with each new decoding iteration, which gradually improves the estimation of the sent codeword.

Even if LDPC codes came equipped with a class of MP decoding algorithms, a substantial effort had to be made in order to advance our knowledge on iterative decoding techniques. Most of the research on decoding algorithms focused on connections with closely-related areas and the design of practical MP decoders [2]. It worth mentioning here one of the most celebrated works, namely the work of Tanner

[3], who described LDPC codes in terms of sparse bipartite graphs and proposed a more general construction of graph-based linear codes. He also generalized the decoding algorithms proposed by Gallager to this new class of graph-based codes, and gave a unified treatment of decoding algorithms for LDPC and product codes.

The capability of MP decoding algorithms to deal with long block lengths opened the way to Shannon limit. They led to the development of graph-based codes and belief-propagation decoding, closely related to the probabilistic approach to coding devised by Shannon. A detailed survey that traces the evolution of channel coding from Hamming codes to capacity-approaching codes can be found in [4]. It is worth noting that unlike the classical coding approach, in which codes are considered and optimized on an individual basis, in the context of probabilistic coding the goal is to find a family of codes that optimizes the average performance under a given MP decoding algorithm. A decisive contribution was made by Richardson and Urbanke [5], who derived a general method for determining the correction capacity of LDPC codes under MP decoding algorithms. They introduced new ensembles of LDPC codes and showed that in the asymptotic limit of the block length, almost all codes (in the same ensemble) behave alike and exhibit a threshold phenomenon, separating the region where reliable transmission is possible from that where it is not. This made possible the design of *irregular* LDPC codes that perform very close to the Shannon limit [6]. Nowadays, LDPC codes are known to be capacity approaching codes for a wide range of channel models, which motivated the increased interest of the scientific community over the last 15 years and supported the rapid transfer of this technology to the industrial sector.

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# Recent Advances on Error Correction Coding with non-binary LDPC Codes

David Declercq

*ETIS laboratory, Cergy-Pontoise, France*

*E-mail:declercq@ensea.fr*

## Abstract

In this tutorial presentation, the iterative decoding techniques for non-binary LDPC codes will be presented, and their efficient implementation discussed. Firstly, the main differences between iterative BP decoding of binary and non-binary LDPC codes will be highlighted., and then the recent solutions proposed in the literature to reduce the complexity of non-binary decoders will be presented. A special focus will be made on the Extended Min-Sum (EMS) algorithm and its efficient implementation. The most recent generalization of the EMS, called "trellis-EMS" (T-EMS) for reducing the decoding latency, will be also presented.

**Key words:** non-binary LDPC decoding, EMS, trellis-EMS, low-latency

## Synopsis

In this tutorial presentation, the iterative decoding techniques for non-binary LDPC codes will be presented, both from the theoretical aspects of Belief Propagation and its analysis, and from more practical aspects of efficient implementation. In a first part, introduction on error correction coding with LDPC will be presented, and the main differences between iterative BP decoding of binary and non-binary LDPC codes will be highlighted. Then, in a second part, the recent solutions proposed in the literature to reduce the complexity of non-binary decoders, both for memory storage and computational burden reduction, will be presented.

A special focus will be made on the Extended Min-Sum (EMS) algorithm [1,2] and its efficient implementation [3]. In addition to the EMS decoder and some of its competitors (like the Min-Max decoder [5]), we will also present a new way to compute modified configuration sets, using a trellis representation of incoming messages to check nodes. This modification of the EMS algorithm is called "trellis-EMS" (T-EMS) [4]. In the T-EMS, the algorithm operates directly on the deviation space by considering a trellis built from differential messages, which serves as a new reliability measure to sort the configurations. In addition to the trellis representation, we will also introduce a technique based on a non-binary soft syndrome computation, in order to be able to compute all output messages of a check-node in parallel, therefore greatly reducing the decoding latency.



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# Reliable Memories From Unreliable Components: Theory and Connections With Codes On Graphs

**Bane Vasić<sup>1</sup>, Predrag Ivaniš<sup>2</sup>**

<sup>1</sup> *Department of ECE, University of Arizona, Tucson*

<sup>2</sup> *School of Electrical Engineering, University of Belgrade*

*E-mail: <sup>1</sup>vasic@ece.arizona.edu, <sup>2</sup>predrag.ivanis@etf.rs*

## Abstract

In this talk we introduce fault-tolerant memory architecture based on low-density parity check codes and iterative decoders. We also present a theoretical analysis of decoders failures.

**Key words:** Low-density parity check codes, Memory architectures, Unreliable components

## Synopsis

Increased integration factor of integrated circuits coupled with stringent energy-efficiency constraints necessities a new design paradigm for Very Large Scale Integration (VLSI) technologies in which fully reliable operation of hardware components is not guaranteed. In this presentation we discuss an error control coding (ECC) as a method for ensuring fault-tolerance of systems build of unreliable hardware with a special focus on fault-tolerant memories. This approach, in contrast to the widely used von Neumann's triple modular redundancy, was introduced in the late sixties and early seventies by Taylor [1] and Kuznetsov [2], while the equivalence between Gallager B decoder built from unreliable logic gates and Taylor-Kuznetsov fault-tolerant memory architectures was first observed by our research group in [3] and [4], and further developed in [5] into a theoretical framework for analysis and design of faulty decoders of low-density parity check (LDPC) codes.

We start by introducing physical reasons for semiconductor devices failures. These reasons depend on the technology used but can be broadly divided into: permanent, intermittent and transient. We focus on the third type of faults, transient faults (TFs), which also referred to as soft errors and are mainly due to single or multiple event upsets or timing errors. These errors have probabilistic behavior and can be described statistically.

After that, we discuss Taylor's proof that a memory built from unreliable components can achieve storage capacity  $C$  for all memory redundancies greater than  $1/C$ . In the Taylor-Kuznetsov (TK) model, the information is stored in a coded form, i.e., the stored vector is a codeword of some  $(n, k)$  block ECC. The memory in which the bits are stored is also assumed to be unreliable. It is connected to a correcting circuit, which periodically updates the memory using given decoding scheme. The redundancy necessary to ensure memory reliability grows linearly with the memory size. A memory failure occurs if the error pattern in the memory is uncorrectable by a perfect decoder in some maximum number of iterations.

In TK memory architecture the user information is first encoded by a regular binary LDPC code of length  $n$  and dimension  $k$ . The stored codeword, denoted as  $\mathbf{x}=(x_1, x_2, \dots, x_n)$ , consists of  $n$  variable bits  $x_i$ , ( $i=1, \dots, n$ ) which are involved in exactly  $J$  parity-check equations. The  $j$ -th component of the vector  $\mathbf{c}=\mathbf{xH}^T$ , called a syndrome, corresponds to the value of  $j$ -th parity-check sum, and can be satisfied if  $c_j=0$  and unsatisfied if  $c_j=1$ . After encoding, the  $J$  identical copies of every coded bit  $x_i$ ,  $\{x_i^{(1)}, x_i^{(2)}, \dots, x_i^{(J)}\}$  are stored in  $J$  registers. Registers are unreliable. In each iteration, the estimates of each of these copies are obtained by using one combination of  $J-1$  checks by the following steps: (i) calculating the parity checks for each bit-copy (exclude one distinct parity check from the original set of check for each bit-copy), (ii) flipping the value of the bit-copy if majority of the parity checks are unsatisfied. The decision element in this case is a majority logic gate whose output is 1 if half or more of the parity checks are non-zero. We present a theoretical analysis of faulty decoders

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# Fault Tolerant Decoders

Predrag Ivaniš<sup>1</sup>, Bane Vasić<sup>2</sup>

<sup>1</sup>*School of Electrical Engineering, University of Belgrade, Serbia*

<sup>2</sup>*Department of ECE, University of Arizona, Tucson, USA*

*E-mail:* <sup>1</sup>predrag.ivanis@etf.rs, <sup>2</sup>vasic@ece.arizona.edu

## Abstract

In this talk we introduce a novel class of fault-tolerant decoders for low-density parity check codes, based on bit-flipping decoding algorithm. Presented decoding algorithm is not only superior to other decoding algorithms of this type, but also robust to logic gate failures.

**Key words:** Bit flipping, Fault tolerance, Iterative decoders, Low density parity check codes

## Synopsis

According to new design paradigm for Very Large Scale Integration (VLSI) technologies, due to lower supply voltages and variations in technological process, fully reliable operations are not guaranteed in nano-scale devices [1]. A hardware component is assumed to be unreliable if it is subject to so-called transient faults, i.e. faults that manifest themselves at particular time instants but do not necessarily persist for later times [2]. An integral part of many such systems, designed for communications or computing, is error-control coding whose role is to maintain the data integrity. Thus, analysis of different decoding algorithms for low density parity check (LDPC) codes under unreliable hardware is meaningful. The density evolution analysis of sum-product algorithm (SPA) [3] and Gallager B algorithm [4] demonstrate robustness of these algorithms to the transient failures.

The reliable storage of data in a memory built from unreliable logic gates under transient failures can be achieved by employing LDPC codes and simple bit-flipping (BF) decoding [5]. In this talk we focus on modifications of BF algorithms that are suitable if only bit hard decisions are available. Although the performances of BF decoder are typically inferior when compared to the Gallager-B algorithm, we report the decoding algorithm based on BF with significant performance improvement, that has large immunity to the gate failures.

## Fault tolerant decoders for BSC

We start by a overview of hard decision decoders based on BF and Gallager A/B algorithms. The structure of variable node processors will be explained and implementation of these decoders in unreliable hardware will be considered. Further, we will explain Gradient Descent Bit Flipping (GDBF) algorithm [6], where the inverse function is represented in the form that is more suitable for BSC. In this algorithm, the most critical value of the inverse function determines the bits that should be flipped in the current iteration. This algorithm is suitable for hardware implementation as it can be designed by using XOR and ML gates. Then, we propose version of GDBF decoder where the probabilistic mechanism is incorporated in the decoder structure by using generator of uniform random numbers. In this algorithm, the most critical value of the modified function represents only necessary condition for flipping.

We will show that the logic gate failures can improve performances of GDBF decoder, in contrary to classical BF decoders. By using the knowledge about trapping sets, we optimize the probabilistic mechanism to improve the performance of GDBF decoder realized in faulty hardware. It will be shown that the proposed solution outperforms Gallager-B algorithm and have performances compared to more complex message passing algorithms. This is a hard-decision algorithm with the best known performances in the water-fall region on BSC, robust to the failures in logic gates.

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# Memory Efficient APP Decoding of LDPC Codes

Velimir Ilić<sup>1</sup>, Elsa Dupraz<sup>2</sup>, David Declercq<sup>3</sup>, Bane Vasić<sup>4</sup>

<sup>1</sup>*Mathematical Institute SANU, Belgrade, Serbia*

<sup>2,3</sup>*ETIS laboratory, Cergy-Pontoise, France*

<sup>4</sup>*Department of ECE, University of Arizona, Tucson*

*E-mail:* <sup>1</sup>velimir.ilic@gmail.com, <sup>2</sup>dupraz@ensea.fr, <sup>3</sup>declercq@ensea.fr, <sup>4</sup>vasic@ece.arizona.edu

## Abstract

We propose memory efficient parallel and serial iterative a posteriori probability (APP) decoders. They require memory that is linear in the number of nodes in the Tanner graph of the code. For high-rate codes this is a significant saving compared to the existing iterative decoders, which require memory that is at least proportional to the number of edges. We present a framework for theoretical analysis of the decoders and show that the serial version achieves a lower level of error probability than the parallel one, for a small number of iterations.

**Key words:** BP decoder, APP decoder, LDPC, memory complexity, error probability analysis

## Synopsis

Belief propagation (BP) is an iterative message-passing algorithm for decoding low density parity check (LDPC) codes [2], widely used in many systems. Despite its good error correction performances and capability of approaching the Shannon limit, BP suffers from large memory requirements for message processing and storage, proportional to the number of edges in the Tanner graph of the code [6]. Such large memory requirements coupled with additional hardware resources needed for the message updating make the BP less attractive in applications with stringing constraints on decoding throughput and code rate.

A posteriori probability (APP) decoder [1] is a suboptimal alternative to BP, in which the variable node processing is simplified by allowing variables to send messages in an intrinsic manner, and a message from a variable node corresponds to a posteriori value used to estimate that variable. While overall computational savings introduced by the APP decoder are not significant, as the BP variable node processing is already simple, the computations can be realized in a memory efficient way.

We propose two memory-efficient versions of APP decoder based on parallel scheduling [3] and serial scheduling [7], [4] schemes, previously considered for BP decoders. They both require memory proportional to the number

of nodes in the Tanner graph of the LDPC code, rather than to the number of edges, as proposed in [1]. We consider the theoretical analysis for the parallel APP decoder proposed in [5], and adapt it to the case of serial scheduling. According to our theoretical analysis, we show that, for a small number of iterations, serial scheduling enables to achieve a lower level of error probability than parallel scheduling.

## Acknowledgment

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# Spherical coverage construction and verification

Marko D. Petković<sup>1</sup>, David (Dragoljub) Pokrajac<sup>2</sup>, Nenad Živić<sup>3</sup>

<sup>1,3</sup>*Faculty of Sciences and Mathematics, University of Niš, Višegradska 33, Serbia*

<sup>2</sup>*Computer and information sciences department, Delaware State University, Dover, DE, USA*

*E-mail:* <sup>1</sup>dexterofnis@gmail.com, <sup>2</sup>dpokrajac@desu.edu, <sup>3</sup>nenad.zivic@pmf.edu.rs

## Abstract

We consider the problem of construction and verification of the coverage of  $d$ -dimensional unit hypersphere by the spherical hypercaps. These problems has direct application in reverse  $k$ -nearest neighbor search, covering a globe by radial transmitters and coding theory. The verification problem assumes that given a set of hypercaps by their central point  $t_i$  and radius  $\alpha_i$  ( $i = 1, 2, \dots, n$ ), one has to check whether those hypercaps completely cover the unit sphere. The construction problem consider the construction of the covering set of  $n$  cones with equal angle  $\alpha$ , such that  $\alpha$  is minimal. A heuristics based on the potential minimization is shown and compared with the referent results for  $d = 3$ . This is used for generation of the higher dimensional covering constellations. Finally, we present the Voronoi diagram based approach for covering construction. It is shown that the required angle is constantly decreasing throughout the method, implying that a method certainly reaches local minimum.

**Key words:** hypercaps, coverage, iterative methods, Voronoi diagram.

## Synopsis

Covering of the (hyper)sphere by the (hyper)cones is the problem which arises in many different applications. The first and the most direct is covering of the Earth by a set of omnidirectional transmitters. The reverse  $k$ -nearest neighbor algorithm, given in [4], needs the division of the space by cones, in order to reduce the number of necessary  $k$ -nearest neighbor calls. Finally, it has also the applications in a coding theory, for the construction of constellations used in joint source–channel coding. A similar problem for polyhedral cones was also recently considered in [2].

The verification problem can be defined as follows: Check whether the given set of  $d$ -dimensional (hyper)cones  $C_1, \dots, C_n$  defined by:

$$C_i = C(t_i; \theta_i) = \left\{ x \in \mathbb{R}^d \mid \frac{(x, t_i)}{\|x\| \|t_i\|} \geq \theta_i \right\}.$$

covers the unit (hyper)sphere  $S_d(1)$ . A problem will be denoted by **SphCovVer**. Equivalently, one can consider a (hyper)caps  $K_i = S_d(1) \cap C_i$  and check whether they cover  $S_d(1)$ . The parameter  $\theta_i$  is given by  $\theta_i = \cos(\alpha_i/2)$  where  $\alpha_i$  is the angle of the cone  $C_i$ .



It is shown that the problem **SphCovVer** is NP-hard. A recursive algorithm **Cover**, based on the dimension reduction and inversion, is then constructed. It is shown that its time complexity is  $\mathcal{O}(n^{d-1} \log n)$  and it can be used also for the uncovered point computation, when the constellation is not covering. Numerical results show that the constructed algorithm possesses a good performance in practice and almost never reach its theoretical complexity.

In the coverage construction problem, one need to construct the covering constellation of a total  $n$  **equal** hypercones having the minimal angle  $\alpha$ . Equivalently, for a given angle  $\alpha$ , one have to find a minimal number of covering cones  $n$  with the angle  $\alpha$ . It is clear that a centers of such constellation should be "uniformly" displaced over the unit hypersphere. One class of methods for uniform distribution of points on the sphere is the minimization of the function:

$$I(t_1, t_2, \dots, t_n) = \sum_{1 \leq i < j \leq N} \phi(\|t_i - t_j\|).$$

where  $\phi$  is corresponding potential function. We used a Forces methods shown in [1] to solve the previous minimization problem and the results for  $d = 3$  are compared with the referent ones given in [3]. The minimal angle for finally obtained constellation is computed using the algorithm **Cover**. Due to the excellent agreement in that case, we give the results for the higher dimensions.

At the end, we show the construction method based on Voronoi diagrams which will probably produce the constellations with the best possible minimal angle.

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# Challenges of Cryptology for Cyber-Security

**Miodrag Mihaljević**

*Mathematical Institute, Serbian Academy of Sciences and Arts, Kneza Mihaila 36, Belgrade, Serbia*

*E-mail: miodragm@turing.mi.sanu.ac.rs*

## Abstract

This talk discusses certain issues of security evaluation and design of advanced cryptographic techniques for cyber-security.

**Key words:** Cyber-Security, Lightweight Cryptography, IoT security, M2M communications security, Cloud security

## Synopsis

In order to support further expansion of Cyber-space on the benefits of the entire society, and in order to avoid disastrous impacts of misuse of the Cyber-space, we face the following two issues: (i) Extensive employment of cyber-security mechanisms; (ii) Overheads to the main functionality implied by employed security mechanisms. Consequently, Fig. 1 points out two main concerns regarding cryptographic techniques to be employed for cyber-security mechanisms.

## Challenges

According to Fig. 1, we point out to the following two challenges: (i) Requirement for provable security of employed security mechanisms within a given security evaluation scenario, and (ii) Requirement for minimization of the overheads implied by the security mechanisms. Certain goals are illustrated in Fig. 2.

## Acknowledgment

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# Scalable Video Delivery for the Future Internet

Zoran Bojkovic<sup>1</sup>, Bojan Bakmaz<sup>2</sup>

<sup>1</sup>*University of Belgrade, Studentski trg 1, Belgrade, Serbia*

<sup>2</sup>*Faculty of Transport and Traffic Engineering, University of Belgrade, V. Stepe 305, Belgrade, Serbia*

*E-mail: <sup>1</sup>z.bojkovic@yahoo.com, <sup>2</sup>b.bakmaz@sf.bg.ac.rs*

## Abstract

Although future networks will require a multimedia transport solution that is more aware of a delivery network's requirements, Future Internet (FI) is expected to be more agile, scalable, secure and reliable. On the other hand, scalable video delivery seems to be the key for efficient streaming in FI applications. The adaptation is performed in the compressed domain by directly removing parts of the bitstream. This paper seeks to provide the concept of scalable video delivery as a key for efficient streaming of FI. After real-time streaming of video content presentation, MPEG-DASH standard is invoked. Next, streaming in content-aware networks is analyzed. Wavelet-based scalable video coding (SVC) together with event-based scalable coding conclude the presentation.

**Key words:** Future Internet, MPEG-DASH, Scalable video delivery, Wavelet-based SVC

## Synopsis

Most of Internet traffic is due to video content. Thus, efficient video coding and streaming are of great importance. Contrasting the traditional client-server model, in peer-to-peer (P2P) distribution models, video is delivered to the end users not directly from the servers, but by converting users to content redistributors. In scalable video coding, the signal is separated into multiple layers of different visual importance. The base layer can be independently decoded and it provides basic video quality. The enhancement layers can only be decoded together with the base layer and these further refine the video quality [1].

If video content is encoded in a scalable way, it can be adapted to any required spatio-temporal resolution and quality in the compressed domain, according to peers' available bandwidth and end-user context requirements. Enhancements of layered scalable coding have been proposed to provide further

granularity scalability [2]. Scalable video representation provides fast adaptation to bandwidth variations, as well as inherent error resilience and complexity scalability properties that are essential for efficient transmission over error prone wireless networks.

Having in mind the fact that spectrum is a limited resource, and allocating new bands to streaming services is a long and expensive process. Increasing the spectral efficiency per link is approaching its limits, i.e., fourth generation mobile systems such as Long Term Evolution (LTE) have a near-optimal physical layer using orthogonal frequency division multiplexing (OFDM) together with capacity-approaching codes and multiple antenna elements.

In this paper, we start with real-time streaming of video content. Next, Hypertext Transport Protocol (HTTP) together with MPEG Dynamic Adaptive Streaming (DASH) will be presented. Streaming in content-aware networks is pointed out, too. Scalable video streaming including wavelet-based scalable video coding (W-SVC), as well as event-based scalable coding conclude the article.

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# Application of Extended Huffman Coding on Three-Level Scalar Quantizer for Gaussian Source

Lazar Velimirović<sup>1</sup>, Zoran Perić<sup>2</sup>, Bojan Denić<sup>2</sup>

<sup>1</sup>*Mathematical Institute of the Serbian Academy of Sciences and Arts, 11001 Belgrade, Serbia*

<sup>2</sup>*Faculty of Electronic Engineering, University of Nis, 18000 Nis, Serbia,*

*E-mail:* <sup>1</sup>lazar.velimirovic@mi.sanu.ac.rs, <sup>2</sup>zoran.peric@elfak.ni.ac.rs, <sup>2</sup>bojandenic4@gmail.com

## Abstract

This paper presents design process of the optimal three-level scalar quantizer with extended Huffman coding. The optimal value of the decision threshold is achieved by optimization of the decision threshold depending on the maximum value of signal to quantization noise ratio. The calculation of average bit rate and source entropy, as well as the comparison between these parameters is done. The convergence of the average bit rate to the source entropy is thoroughly analysed for two, three, and four symbol blocks. Performances of the proposed quantizer in terms of signal to quantization noise ratio are estimated by comparing numerical values to the quantizer having two quantization levels for the Gaussian probability density function. Also, analysis of the average bit rate and the source entropy depending on the size of symbol blocks is performed.

**Key words:** Quantization, Scalar quantizer, Huffman coding, Entropy coding

## Synopsis

Shannon's First Theorem answers what performances can be achieved with statistical coding. Furthermore, this theorem gives an answer on how these performances can be achieved with sufficient source expansion. However, it does not indicate clearly how to achieve a compact code for the fixed source without further expansion [1]. Huffman coding procedure, which is discussed in the paper, offers a solution. Huffman coding enables direct determination of the compact code [1].

This paper considers a symmetrical model of the three-level scalar quantizer, with level zero representation, on which an extended Huffman coding applies. Extended Huffman coding technique is the procedure which determines the optimal length of codewords for the blocks of two or more symbols [2,3]. In this paper, results obtained for the blocks of two, three, and four symbols are shown and analysed. The optimal value of the decision threshold is numerically determined, respecting the criterion of maximum signal to quantization noise ratio (SQNR). The optimal decision threshold value of the symmetrical scalar quantizer with number of levels  $N = 4$ , for Gaussian probability density function, equals to  $t_1 = -t_2 = 0.982$ . That is why, the optimal decision threshold value of the proposed quantizer is determined within the range of  $t_1 = -t_2 = 0.1$  to  $t_1 = -t_2 = 0.8$ . For the optimal decision threshold value, the average bit rate and the source entropy for the blocks of two, three, and four symbols are calculated.

On the basis of the results obtained it can be noticed that better performances can be achieved with the proposed quantizer model than with the two-level scalar quantizer model for the same PDF at the input. Also, it can be concluded that the average bit rate converges more to the source entropy for the blocks of four symbols than for the blocks of two symbols. However, as the number of symbol blocks increases, the complexity of the proposed model increases, too. Therefore, the optimal solution regarding the complexity of the proposed model and achieved SQNR value is the proposed scalar quantizer for the blocks of three symbols.

## Acknowledgment

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# An Efficient Code Converter for Absolute Position Encoders

Milan Dinčić<sup>1</sup>, Dragan Denić<sup>2</sup>, Goran Miljković<sup>3</sup>, Aleksandar Jocić<sup>4</sup>, Jelena Lukić<sup>5</sup>

<sup>1,2,3,4,5</sup>*Faculty of Electronic Engineering, University of Niš, Serbia, Aleksandra Medvedeva 14, 18000 Niš*

*E-mail:* <sup>1</sup>mdincha@hotmail.com, <sup>2</sup>dragan.denic@elfak.ni.ac.rs, <sup>3</sup>goran.miljkovic@elfak.ni.ac.rs,  
<sup>4</sup>aleksandar.jocic@elfak.ni.ac.rs, <sup>5</sup>jelena.lukic@elfak.ni.ac.rs

## Abstract

This paper presents an efficient method for realization of pseudorandom/natural code conversion for absolute position encoders. The method is based on the Galois code converter, where XOR logic gates in the feedback configuration are connected in parallel and hence there is no time delay accumulation. Galois code converter is much faster than Fibonacci code converter where XOR logic gates in feedback configuration are connected serially, which produces the accumulation of time delays. Both Galois and Fibonacci converters are designed based on generator polynomials. Galois code converter requires an initial logic for proper functioning. The design of this logic, which is the main task during the designing of Galois converter, will be described in this paper.

**Key words:** absolute position encoders, pseudorandom binary sequences, pseudorandom/natural code converters, Fibonacci code converters, Galois code converters

## Synopsis

Measurement of absolute position (linear and angular) is very important in many applications where linear or angular movement is involved. One very common method for the position measurement is the using of absolute position encoders. Very important property of these encoders is the fact that they give digital output, i.e. there is no need for an additional A/D (analog-to-digital) conversion. Significant progress in the area of absolute position encoders was introduction of absolute position encoders with pseudorandom binary sequences (PRBS), since these converters can achieve much higher resolution (the number of bits used for coding of one position). These absolute position encoders with PRBS are called pseudorandom position encoders.



However, there is one problem with pseudorandom encoders – pseudorandom code is not appropriate for using in digital electronics. Hence, a conversion from pseudorandom to the natural binary code has to be done. Fibonacci pseudorandom/natural code converters was used firstly [1, 2, 3, 4, 5]. The problem with Fibonacci converters is the fact that XOR logic gates in feedback configuration are connected serially, which produces accumulation of time delays of these XOR gates. This leads to the high overall delay of Fibonacci converter, which slows down the entire pseudorandom encoder.

In this paper we propose the way how to improve the speed of pseudorandom/natural code conversion. The idea is to use Galois code converters [6] instead of Fibonacci code converters. In Galois code converter XOR logic gates in feedback configuration are connected in parallel. Hence, there is no accumulation of time delays of XOR gates, i.e. the overall delay of the whole feedback loop is equal to the delay of only one XOR gate. In this way, the speed of pseudorandom/natural code converter increases and therefore the entire pseudorandom position encoder is much faster and efficient.

## Acknowledgment

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# DPCM System With Polar Quantizer for Source Coding of Measurement Signals With Gaussian Distribution

Milan Dinčić<sup>1</sup>, Zoran Perić<sup>2</sup>, Aleksandar Jocić<sup>3</sup>, Dragan Denić<sup>4</sup>, Dragan Radenković<sup>5</sup>

<sup>1,2,3,4,5</sup>*Faculty of Electronic Engineering, University of Niš, Serbia, Aleksandra Medvedeva 14, 18000 Niš*

*E-mail:* <sup>1</sup>mdincha@hotmail.com, <sup>2</sup>zoran.peric@elfak.ni.ac.rs,  
<sup>3</sup>aleksandar.jocic@elfak.ni.ac.rs, <sup>4</sup>dragan.denic@elfak.ni.ac.rs, <sup>5</sup>dragan.radenkovic@elfak.ni.ac.rs

## Abstract

A new model for source coding of measurement signals with Gaussian distribution is proposed in this paper. The model consists of DPCM (differential pulse code modulation) system and polar quantizer. Double benefit can be achieved using this model: due to DPCM system we can exploit correlation of measurement signals while due to polar quantizer we can achieve high value of SQNR (signal-to-quantization noise ratio).

**Key words:** DPCM system, polar quantization, Gaussian distribution, source coding

## Synopsis

Measurement signals are mostly analog, while modern systems for processing and transmission of signals are digital. Hence, an analog-to-digital (A/D) converter is a crucial part of almost all modern measurement systems. Quantizer is the most important part of any A/D converter. Beside A/D conversion, compression of signals is also very important due to limited resources for signal transmission and storage (channel bandwidth and memory space). Compression also can be achieved with the good choice of quantizer. Measurement signals are random in nature and can be modelled with some statistical distribution. Due to the central limit theorem, the most of measurement signals are modelled with the Gaussian distribution. Therefore, quantizers designed for Gaussian distribution will be considered in this paper.

There are two main types of quantizers: scalar and vector [1, 2]. In scalar quantizers each sample of the signal is separately quantized, while in vector quantizer  $n$  samples ( $n \in \mathbb{N}$ ,  $n > 1$ ) are jointly

quantized. Parameter  $n$  represents the dimension of the vector quantizer. Vector quantizers can achieve better performances than scalar quantizers, i.e. they give higher SQNR for the same bit-rate. On the other hand, complexity of vector quantizers is much higher compared to scalar quantizers and it increases with the increasing of dimension  $n$ . Two-dimensional vector quantizers ( $n = 2$ ) are the most used vector quantizers since they are the simplest of all vector quantizers but they still have much better performances than scalar quantizers. For memoryless signal with the Gaussian distribution it is much easier to design two-dimensional vector quantizer in polar than in Cartesian coordinates and such quantizers are called polar quantizers [3, 4, 5, 6].

Measurement signals are highly correlated, which can be exploited for compression. However, polar quantizers do not use signal correlation since they are designed for memoryless signals. Signal correlation can be very successfully exploited by DPCM system [1].

In this paper we propose a new model which is a combination of DPCM system and polar quantizer. This is a new approach in the source coding theory. Until now, DPCM system was used with scalar (not with vector) quantizers and vector quantizers are used without DPCM systems. With this new model we have double benefit: we exploit signal correlation using DPCM system and we increase SQNR using polar quantization. Hence, this model can achieve better performances compared to previously used models.

## Acknowledgment

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# On the Implementation of Finite Field Operations on CPUs and GPUs

Dušan B. Gajić<sup>1</sup>, Radomir S. Stanković<sup>1</sup>

<sup>1</sup>*CIITLab, Dept. of Computer Science, Faculty of Electronic Engineering, University of Niš*

*Aleksandra Medvedeva 14, 18000 Niš, Serbia*

*E-mail: <sup>1</sup>dusan.b.gajic@gmail.com, <sup>1</sup>radomir.stankovic@gmail.com*

## Abstract

We consider the impact of different techniques for the implementation of finite field operations used in computing Galois field (GF) transforms on the performance of corresponding programs processed on central processing units (CPUs) and graphics processing units (GPUs). On CPUs, using lookup tables (LUTs) is found to be faster than applying modulo operators from the C/C++ programming language, especially in the case of Galois fields of prime order. However, on GPUs, modulo operators from CUDA and OpenCL should be used whenever possible, since they offer significantly shorter processing times than LUTs.

**Key words:** Finite fields, Galois field transform, modular arithmetic, software implementations, GPU computing.

## Synopsis

Performing computations in finite (Galois) fields of order  $p$ , denoted as  $GF(p)$ , is an important task in several areas of computer science and engineering, i.e., information theory, coding theory, communications, and signal processing [7]. The finite field operations used in these computations can be implemented in different ways, depending on the order of the field. When  $p$  is a prime number, the field operations are the addition and multiplication modulo  $p$  [7]. In this case, we can use modulo  $p$  arithmetic operators, usually available in high-level programming languages, or we can implement the operations using lookup tables (LUTs). For  $p$  a non-prime number, the finite field operations differ from the modulo  $p$  arithmetic operators, and, therefore, we are confound to implementing them using LUTs.

In the presented research, we discuss the impact of the implementation of finite field operations on central processing units (CPUs) and graphics processing units (GPUs) on the performance of fast Cooley-Tukey algorithms for computing Galois field (GF) transforms [2, 3]. For the development of CPU implementations we use the C++ programming language. The GPU implementations are developed using both Nvidia CUDA [6] and OpenCL [4]. It

is found that the way of implementing finite field operations has a considerable impact on the overall performance of fast Cooley-Tukey algorithms for  $GF$ -transforms. This impact is especially strong for Galois fields of prime order  $p$ . The efficiency of the implementation of finite field operations is found to be highly dependent on the characteristics of the used computing platform and programming framework [1, 5].

Recommendations for practical implementation of finite field operations on CPUs and GPUs can be summarized as follows. When using the Cooley-Tukey algorithms for performing the  $GF$ -transform in a finite field of prime order  $p$  and on CPUs, LUTs are the method of choice for implementing modulo  $p$  arithmetic operations, since they offer superior performance. If  $p$  is non-prime and not a power of 2, LUTs are once more preferred. In the case when  $p$  is a power of 2, the modulo operation can be performed using bitwise operations and the performance advantage of using LUTs disappears. For the case of computing on GPUs, we can conclude that modulo  $p$  operations in OpenCL and, especially, CUDA, are implemented and executed in an efficient manner. This makes them highly suitable for computing in finite fields of prime order. The application of GPUs is justified when computing the  $GF$ -spectra of multiple-valued logic (MVL) functions with the number of variables  $n > 6$ . Advantages of using GPUs instead of CPUs for computations, become considerably higher as the number of variables further increases.

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# Estimation of Phase-Noise PDF using Graphical Processing Unit

Dejan N. Milić<sup>1</sup>, Aleksandar S. Cvetković<sup>2</sup>

<sup>1</sup> *University of Niš, Faculty of Electronic Engineering A. Medvedeva 14, 18000 Niš, Serbia*

<sup>2</sup> *University of Belgrade, Faculty of Mechanical Engineering, Kraljice Marije 16, 11000, Belgrade, Serbia*

*E-mail:* <sup>1</sup>dejan.milic@elfak.ni.ac.rs, <sup>2</sup>acvetkovic@mas.bg.ac.rs

## Abstract

Phase noise presents a serious impairment in various areas of telecommunications. This is especially true for optical telecommunication systems employing coherent detection principle. It is of great importance to have a good method of estimation of probability density function since bit-error rates can be estimated theoretically using this pdf. In the recent years there is strong evolution of the hardware platforms which can deliver significant improvement of flops. In this article we are going to use graphical processing unit and exploit its parallel capabilities in order to estimate probability density function of the phase noise in the reasonable amount of time. The algorithm is implemented on Tesla C2075 with computational capability of 2.0 and graphical device driver TCC, product of NVIDIA company.

**Key words:** Phase noise, Parallel computing, GPU, Probability density function, Bit-error rate

## Synopsis

Phase noise may impair seriously a number of telecommunication systems ranging from wireless communication systems to coherent optical communications. Fokker-Plank equation has been proposed as a rigorous method for the analysis of optical systems impaired by phase noise. Numerical solution of the equation, with integrator-filter impulse response, has been reported by Garret et al. in [3]. Various methods have also been proposed over the years in order to solve the equation or approximate its solution, for example [5, 6].

When phase-noisy lightwave of duration  $T$  is on input of a filter with equivalent impulse response  $h(t)$ , response at any given time  $\tau$  is represented as [3]:

$$s(\tau) = \int_0^T h(\tau - x) e^{j\varphi(x)} dx.$$

Phase-noise process  $\varphi(t)$  is a continuous path Brownian motion with zero mean and variance  $E(\varphi^2(t)) = 2\pi\Delta\nu t$ , where  $\Delta\nu$  denotes the laser linewidth.

For the integrator-filter, impulse response is represented as:

$$s(\tau) = \int_0^\tau h(\tau - x) e^{j\varphi(x)} dx,$$

because we assume  $h(\tau) = 0$ , for  $\tau < 0$  or  $\tau > T$ . By substituting  $\tau - x = y$ , after we assume zero-phase initial condition  $\varphi(0) = 0$ , without the loss of generality, phase process  $\varphi(\tau - y)$  will have same statistical properties as the process  $\varphi(\tau) - \varphi(y)$ , and the filter response will be statistically equivalent to [3]:

$$s(\tau) = \int_0^\tau h(y) e^{j(\varphi(y) - \varphi(\tau))} dy.$$

In order to estimate pdf at the given time instance  $t$ , we construct the ensemble of realizations of the random variable  $s(t)$ . The obvious way to exploit parallelism is to simulate one member of the ensemble on the one thread of GPU (see [2, 4]). Since there are maximally 1024 threads per block and the maximum value of blocks supported in one kernel launch is limited to 65536, in one kernel launch we are able to generate an ensemble of around 65 million realizations. The number of allocated register memory per thread is not an issue since the computation of  $s(t)$  can be performed easily using Ito definition of stochastic integration [1]

$$s(\tau) = \sum_{k=0}^{N-1} h(t_k) e^{j(\varphi(t_k) - \varphi(\tau))} \Delta t_k = \sum_{k=0}^{N-1} h(t_k) e^{-j\sqrt{2\pi\Delta\nu(\tau-t_k)}\psi_k} \Delta t_k,$$

where  $t_k \in [0, \tau]$ ,  $k = 0, \dots, N$ ,  $t_k < t_{k+1}$ ,  $k = 0, \dots, N - 1$ , and  $\psi_k$ ,  $k = 0, \dots, N - 1$ , are Gaussian random variables of zero mean and unit variance. For the construction of the Gaussian random variables we use `curand` function from CUDA toolkit. After construction of the ensemble, we use `thrust` library from the `and` function `thrust::sort` routine to sort realization. On the sorted ensemble we calculate distribution using simple counting of the realization smaller then given fixed number, and we use numerical differentiation to computer probability density function from the distribution function.

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# On Performance of Hash Functions

Edin Mulalić<sup>1</sup>, Miomir Stanković<sup>2</sup>, Radomir Stanković<sup>3</sup>

<sup>1</sup>Mathematical Institute of the Serbian Academy of Sciences and Arts, Kneza Mihaila 36, 11000 Belgrade, Serbia

<sup>2</sup>University of Niš, Faculty of Occupational Safety, Čarnojevića 10a, 18000 Niš, Serbia

<sup>3</sup>University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia

E-mail: <sup>1</sup>edinmulalic@yahoo.com, <sup>2</sup>miomir.stankovic@gmail.com, <sup>3</sup>radomir.stankovic@gmail.com

## Abstract

Modern computer science is often faced with analysis and processing of large amount of data. As a consequence of these trends, even though computational power rises with significant rate, search for efficient mechanisms for storing and retrieving data still remains as one of the central topics in research community. Particularly significant methods are hash-based data structures and algorithms, including different types of hash tables [1], Bloom filters [2], hash-encryption [3], various data-streams analysis algorithms [4, 5, 6, 7], and many others.

**Key words:** hash function; universal hashing; entropy; worst-case improvement

## Synopsis

Considering that hashing necessarily includes *randomness*, analysis of performance of hash-based approaches is a subject of study in various branches of mathematics, such as statistics, number theory, and combinatorics. Traditionally the analysis was performed under some simplistic assumptions which made the whole process more mathematically feasible, but at the same time, left a gap between theoretically guaranteed performances and practical observations. By removing idealistic assumption of a perfectly random hash function and introducing the concept of *universal hashing* (or *k-wise independence*), authors of [8] provided a framework for construction and analysis of hash-based structures with strong performance guarantees [9, 10, 11, 12, 13, 14, 15]. However, it should be noted that hash functions with strong theoretical guarantees are often overly complex to be used in practice [16].

Achieving a good compromise between simplicity and efficiency of hash function calculation and a good distribution of low-entropy data is among main challenges in construction of efficient hash functions. Although it is shown that there exist data sequences for which universal hashing does not provide good performances, such cases are widely assumed as “pathological” [9]. In other words, worst-case analysis is considered as impractical and average-case analysis is utilized. When randomness of data is combined with randomness of hash functions, it is highly



unlikely in real life scenario to encounter the worst case, and that is the main strength of this approach. However, the question remains – *what if we do encounter such unlikely situation?* What are our options to deal with this problem? Additionally, how can we exploit more complex hash functions by improving speed of their evaluation? In this talk, we will address these issues, investigate worst-case scenario for hashing of data with unknown underlying probability distribution and, finally, propose a possible strategy for overcoming some of the difficulties.

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# Excitable Schrodinger's Metamedium for Image Processing

V.G. Labunets<sup>1</sup>, S.Murga<sup>1</sup>, E.Ostheimer<sup>2</sup>

<sup>1</sup> Ural Federal University named after the first President of Russia B.N.Yeltsin

<sup>2</sup> Capricat LLC 1340 S. Ocean Blvd., Suite 209 Pompano Beach, 33062 Florida, USA

## Abstract

In this work, we apply quantum cellular automata (QCA) to study pattern formation and image processing in quantum-diffusion Schrodinger systems (QDS)

$$\frac{\partial w}{\partial t} = D \left( \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) + f(x, y, t) \quad (1)$$

with real, imaginary, complex, quaternion and hypercomplex diffusion coefficients  $D$ , where  $f(x, y, t)$  and  $w(x, y, t)$  are input and output images, respectively.

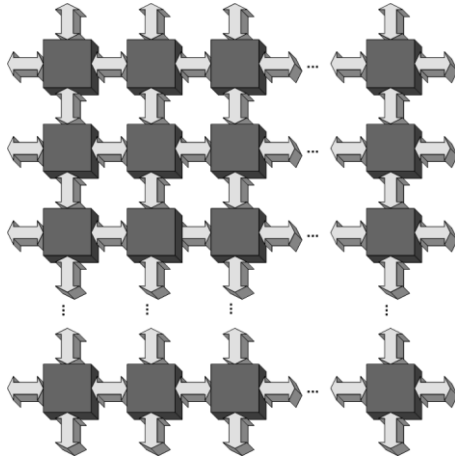
**Key words:** Image Processing, Quantum Cellular Automata, Quantum-Diffusion Schrodinger systems, Metamedium, Hyperspectral Image Processing

## Synopsis

Discretization of (1) gives

$$w^{k+1}(n, m) = w^k(n, m) + D\tau [w^k(n+1, m) + w^k(n-1, m) - 4w^k(n, m) + w^k(n, m+1) + w^k(n, m-1)] / h^2.$$

In this form QDS can be considered as "lattice-based metamaterial models" with various exotic physical parameters. If a traditional computer is thought of as a "programmable object", QDS in the form of QCA is a computer of new kind and is better visualized as a "programmable material".



The purpose of this work is to introduce new metamedium in the form of cellular automata. The cells are placed in a 2D array and they are capable of performing basic hypercomplex operating and exchanging messages about their state. Cellular automata like architectures have been successfully used for computer vision problems and hyperspectral image processing.

# Synthesis of All-Pass Functions for Signal Compression and Expansion

Miloš Djurić<sup>1</sup>, Goran Stancić<sup>2</sup>

<sup>1</sup>*Mathematical Institute SANU, Kneza Mihajla 36, 11000 Belgrade, Serbia*

<sup>2</sup>*University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia*

*E-mail:* <sup>1</sup>djura042@gmail.com, <sup>2</sup>goran.stancic@elfak.ni.ac.rs

## Abstract

System for signal compression and expansion of can be achieved by implementation of some mathematical methods that are given here and have various implementation. These methods can be realized by using parallel connection of two all-pass functions. Proposed algorithm is based on approximation of quadratic function  $a\omega^2 + b\omega$ , where for  $a > 0$  obtained network can be used for signal compression and for  $a < 0$  the network can be used for signal expansion. Concerned algorithm can also be expanded so the degree of compression and the degree of expansion have equal values. Hereof, described system realized by proposed method can find direct application in audio and radar systems.

**Key words:** Signal compression and expansion, All-pass function, Quadratic phase, Complementary network, Signal processing

## Synopsis

Feature of all-pass function are that in Euler's representation its magnitude equals one, and its argument

$$\varphi(\omega) = n\omega - 2\arctg \frac{\sum_{i=0}^n a_i \sin(i\omega)}{\sum_{i=0}^n a_i \cos(i\omega)}$$
. Two all-pass functions that constitute required network are given

by  $H_1(z) = z^n \frac{\sum_{i=0}^n a_i z^{-i}}{\sum_{i=0}^n a_i z^i}$  and  $H_2(z) = z^m \frac{\sum_{i=0}^m b_i z^{-i}}{\sum_{i=0}^m b_i z^i}$ , and network that represent parallel connection

of given all-pass functions  $F(z) = \frac{1}{2} (H_1(z) + (-1)^p H_2(z))$ ,  $p = 0, 1$  [1,2]. Coefficients  $a_i$  of function  $H_1(z)$  are chosen in such a way that its argument  $\varphi_1(\omega)$  approximates quadratic function  $a\omega^2 + b\omega$ . Hitherto, papers in existing literature refer to linear function approximation [2], and these are the first results of the quadratic function approximation. On the other hand, coefficients  $b_i$  of the function  $H_2(z)$  are calculated in small number of iterative cycles in such a way that its arguments approximate function  $\varphi_1(\omega)$ . Procedure for all-pass parallel network coefficients is based on solving linear equations in just a few iterative steps.

First derivative of all-pass function arguments with respect to frequency is in field of filter design called group delay. In case of equal group delay slope, linearly decreasing (for  $a > 0$ ) and linearly increasing (for  $a < 0$ ), complementary filters can be obtained with same values of signal compression and expansion.

This configuration has many desirable characteristics such as computational efficiency [3,4], low sensitivity of constructed filters, and others.

## Acknowledgment

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# Systematic Approach to Nonlinear Filtering Associated With Aggregation Operators

V.G. Labunets <sup>1</sup>, J.Astola <sup>2</sup>, E.Ostheimer <sup>3</sup>

<sup>1</sup>*Ural Federal University named after the first President of Russia B.N.Yeltsin*

<sup>2</sup>*TICSP, Tampere University of Technology, Tampere, Finland*

<sup>3</sup>*Capricat LLC 1340 S. Ocean Blvd., Suite 209 Pompano Beach, 33062 Florida, USA*

## Abstract

The basic idea behind this work is the estimation of the uncorrupted image from the distorted or noisy image, and is also referred to as image “denoising”. To denoise images is to filter out the noise. The challenge is to preserve and enhance important features during the denoising process. For images, for example, an edge is one of the most universal and crucial features. There are various methods to help restore an image from noisy distortions. Each technique has its advantages and disadvantages. Selecting the appropriate method plays a major role in getting the desired image. Noise removal or noise reduction can be done on an image by linear or nonlinear filtering. The more popular linear technique is based on average (on mean) linear operators. Denoising via linear filters normally does not perform satisfactorily since both noise and edges contain high frequencies. Therefore, any practical denoising model has to be nonlinear. In this paper, we propose a new type of nonlinear data-dependent denoising filter called the *aggregation digital filter (ADF)*.

**Key words:** Nonlinear Filtering, Noisy Image, Denoising, Aggregation Operators, Aggregation Digital Filter

## Synopsis

Let us introduce the observation model and notion used throughout this work. We consider noise images of the  $\bar{\mathbf{f}}(\mathbf{x}) = \bar{\mathbf{s}}(\mathbf{x}) + \bar{\boldsymbol{\eta}}(\mathbf{x})$ , where  $\bar{\mathbf{s}}(\mathbf{x}) = s_1(\mathbf{x}), s_2(\mathbf{x}), \dots, s_K(\mathbf{x})$  is the original multichannel signal,  $\bar{\boldsymbol{\eta}}(\mathbf{x}) = \eta_1(\mathbf{x}), \eta_2(\mathbf{x}), \dots, \eta_K(\mathbf{x})$  denotes the multichannel noise introduced into the signal

$\vec{\mathbf{S}}(\mathbf{x})$  to produce the corrupted image  $\vec{\mathbf{f}}(\mathbf{x})$ . Here  $\mathbf{x} = (i, j) \in \mathbf{Z}^2$  (or  $\mathbf{x} = (i, j, k) \in \mathbf{Z}^3$ ) is a 2D (or 3D) coordinates that belong to the image domain and represent the pixel location. If  $\mathbf{x} \in \mathbf{Z}^2, \mathbf{Z}^3$  then  $\vec{\mathbf{f}}(\mathbf{x}), \vec{\mathbf{s}}(\mathbf{x}), \vec{\mathbf{n}}(\mathbf{x})$  are 2D and 3D multichannel images, respectively. The aim of image enhancement is to reduce the noise as much as possible or to find a method which, given  $\vec{\mathbf{S}}(\mathbf{x})$ , derives an image  $\hat{\mathbf{S}}(\mathbf{x})$  as close as possible to the original  $\vec{\mathbf{S}}(\mathbf{x})$ , subject to a suitable optimality criterion. A number of approaches can be found in the literature for solving the image enhancement problem.

In a 2D standard linear and median filters with a square window  $\left[ \mathbf{M}_{(i,j)}(m,n) \right]_{m=-r, n=-r}^{m=+r, n=+r}$  of size  $2r+1 \times 2r+1$  is located at  $(i, j)$  the mean and median replace the central pixel  $\hat{\mathbf{S}}(i, j) = \mathbf{Mean}_{(m,n) \in \mathbf{M}_{(i,j)}} \vec{\mathbf{f}}(m, n)$ ,  $\hat{\mathbf{S}}(i, j) = \mathbf{Med}_{(m,n) \in \mathbf{M}_{(i,j)}} \vec{\mathbf{f}}(m, n)$ , where  $\hat{\mathbf{S}}(i, j)$  is the filtered image,  $\vec{\mathbf{f}}(m, n)_{(m,n) \in \mathbf{M}_{(i,j)}}$  is image block of the fixed size  $2r+1 \times 2r+1$  extracted from  $\vec{\mathbf{f}}$  by moving window  $\mathbf{M}_{(i,j)}$  at the position  $(i, j)$ , **Mean** and **Med** are the mean (average) and median operators.

When those filters are modified as follows  $\hat{\mathbf{S}}(i, j) = \mathbf{Aggreg}_{(k,l) \in \mathbf{M}_{(i,j)}} \vec{\mathbf{f}}(k, l)$ , it becomes an aggregation digital filter, where **Aggreg** is so called *generalized average* or an *aggregation operator*. We show that a large body of non-linear filters proposed to date constitute a proper subset of Aggregation Filters.

# Application of Naive Bayes Classifier in Stereo-Vision Based Object Recognition

Milica Ćirić<sup>1</sup>, Ivan Ćirić<sup>2</sup>, Milan Gocić<sup>3</sup>

<sup>1,3</sup>*Faculty of Civil Engineering and Architecture, Aleksandra Medvedeva 14 Niš, Serbia*

<sup>2</sup>*Faculty of Mechanical Engineering, Aleksandra Medvedeva 14 Niš, Serbia*

*E-mail:* <sup>1</sup>milica.ciric@gaf.ni.ac.rs, <sup>2</sup>ciric.ivan@masfak.ni.ac.rs, <sup>3</sup>milan.gocic@gaf.ni.ac.rs

## Abstract

In this paper, the problem of robot vision system is considered as a part of hierarchically highest level of control that is used for object recognition and that allows adequate and reliable robotic grasping. Thus, the application of a classifier is proposed in the object recognition process for an advanced and robust robot vision system. This technique can be used in 3D object reconstruction with an accurate determination of position and orientation of rigid body that can help in deciding where to grasp an object with a robot arm. Classification is performed using features obtained as a result of image segmentation.

**Key words:** Naive Bayes classification, Robot vision, Image segmentation, Object recognition

## Synopsis

One of the key requirements in the field of service robotics is the robust perception of the robot environment, which is necessary for autonomous object manipulation. A robot vision system is used to robustly analyse the images of complex scenes where the objects to be recognized are surrounded by a variety of other objects. As well as being robust against cluttered scenes, a robot vision system has to be robust against unpredictability in the appearance of objects due to external influences, such as variable illumination.

The main objective of present research is to use both the intelligent segmentation algorithm and Naive Bayes classifier [1] for object recognition. The proposed robust methods are an alternative to the conventional approach of using additional sensors or introducing a more controlled environment.



For development of reliable computationally intelligent object recognition in robotic vision, *the robot system FRIEND*, a 7 Degrees of Freedom manipulator mounted on an electrical wheelchair with Bumblebee® stereo-camera system is used [2, 3]. The user navigates the system in front of the container containing objects related to the particular working scenario (for example, fridge in the “serving a drink” working scenario). The stereo camera views the scene in front of the robotic system including the container with the objects and the manipulator. Obtained stereo images are processed in a sequence of image processing operations aiming at the extraction of features needed for both 2D object recognition in stereo images and 3D stereo reconstruction of the manipulator’s environment. To achieve good object segmentation it is necessary to adjust the object segmentation interval as illumination conditions change. Finding the optimal object segmentation interval that provides a segmented object image of good quality, appropriate for subsequent object feature extraction for object recognition and reconstruction, was done using computationally intelligent algorithm based on neural networks. A Naive Bayes classifier [4] is then used for classifying segmented objects. The premise is that each feature of an instance is completely unrelated any other feature of the same instance, i.e. probability that an instance belongs to a certain category is influenced by each feature independently. Characteristics used as features for object classification are proportionality and connectivity, which are both direct results of the segmentation process. In this study, we used Mallet [5] and its built-in functionalities of Naive Bayes implementation, for statistical natural language processing, document classification, clustering, topic modeling, information extraction, and other machine learning applications to text.

## Acknowledgment

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# Modeling of the Genetic Code

Branko Dragovich<sup>1</sup>, Nataša Ž. Mišić<sup>2</sup>

<sup>1</sup>*Institute of Physics, University of Belgrade, Pregrevica 118, 11080 Belgrade, Serbia*

<sup>2</sup>*Lola Institute, Kneza Višeslava 70a, Belgrade, Serbia*

*E-mail:* <sup>1</sup>dragovich@ipb.ac.rs, <sup>2</sup>nmisic@afrodita.rcub.bg.ac.rs

## Abstract

The genetic code is something what plays central role in all living organisms at our planet Earth. It is a perfect information phenomenon. It is experimentally deciphered in the 1960s, but still is not quite well theoretically understood. We present and advocate ultrametric approach to modeling of the genetic code. We show that by some  $p$ -adic distances one can quantitatively describe structure of the codon space related to amino acids distribution. It seems that ultrametric approach is simple and natural in description of similarity and nearness not only in the genetic code but also in some other bioinformation systems.

**Key words:** Genetic code, DNA, Codons,  $p$ -Adic numbers, Ultrametricity, Amino acids

## Synopsis

The genetic code (GC) is a mapping from the space of 64 codons onto set of 21 elements, which consists of 20 amino acids and 1 stop signal. Codons are ordered triples of four nucleotides (C, A, T (U), G) and they are building blocks of the genes. These canonical amino acids are building blocks of the proteins, and stop signal terminates synthesis of proteins. The standard (canonical) genetic code, which functions in cells of almost all living organisms, was experimentally deciphered in 1960s and is usually presented as a table of codons with the corresponding amino acids. However, modeling of the possible connection between DNA and proteins started soon after discovery of the DNA double helicoidal structure in 1953. All this time of 60 years there has been permanent interest in modeling of the genetic code. Starting points in description of the GC have been rather different: mathematical, physical, chemical, biological and informational. In some approaches more attention has been paid to structure of the set of amino acids than to the structure of the set of codons (in [1] connection of number of constituents of amino acids and some properties of elementary number theory was discussed), and vice versa.

One of the main reasons for modeling the GC is a huge number (about  $10^{84}$ ) of possible connections between 64 codons and 20 amino acids with one stop signal, while in living organisms there is practically one GC with about thirty slight variations. The GC is not a uniform distribution of three codons onto each of 20 amino acids, but it is still highly non random. Contemporary standpoint on the GC origin and evolution is based on three, to some

extent mutually depended processes: the physico-chemical affinity of its constituents (stereochemical theory), the minimization of the adverse effect of point mutations and translation errors (error minimization theory) and the amino acid biosynthesis pathways (coevolution theory). Hence the task of modeling of the GC should be the quest of a simple and natural description of all of these processes. So far there is no such model which contains complete GC description and some of existing models are rather complicated. In this communication we present an ultrametric model of the GC, which is mathematically simple and conceptually natural, where ultrametricity is quantified by the  $p$ -adic distance.

Let us imagine all codons as an information space in which meaning of codons is determined by their relation to amino acids. Then one can easily observe that meaning of codons is organized in the very similar way as meaning of words in human languages – words with the same root (and different prefixes or suffixes) have very similar meaning. Usually, the roots of codons are the first two letters and two codons with the same root code the same or similar amino acids. This is the reason that the GC can be viewed as a natural language, which consists of 4 letters with 64 three-letter words of 21 meanings.

$p$ -Adic structure of the codon space was introduced in [2] and then discussed in some papers (see, e.g. [3] and references therein). The basics of this approach are as follows. We advocate the idea that functional similarity between codons should be described by some  $p$ -adic distances. To this end, codons are identified with three-digit 5-adic numbers by identifying nucleotides with digits ( $C = 1, A = 2, T = U = 3, G = 4$ ). To differ purines and pyrimidines, we use 2-adic distance. In this way one obtains structure of codon space in the form of doublets in the GC of the vertebrate mitochondria, where each doublet of codons codes either one of 20 amino acids or stop signal. Other genetic codes can be obtained from this one as change of meaning of a few codons. This  $p$ -adic structure of codon space in natural way presents degeneration of the GC.

In this talk, it will be presented a brief review of modeling of the genetic code, emphasizing its inherent ultrametric structure which is quantitatively described by some  $p$ -adic distances. Recall that  $p$ -adic distance is the most important example of the ultrametric distance, which satisfies strong (ultrametric) triangle inequality, i.e.  $d(x, y) \leq \max\{d(x, z), d(z, y)\}$ . It will be also discussed extension of this  $p$ -adic ultrametric approach to amino acids, to modification of the Hamming distance, to possible evolution of the GC and to some other bioinformation systems.

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# Is The Animal Brain A Clifford Algebra Computer?

V.G. Labunets <sup>1</sup>, E.Ostheimer <sup>2</sup>

<sup>1</sup>*Ural Federal University named after the first President of Russia B.N.Yeltsin*

<sup>2</sup>*Capricat LLC 1340 S. Ocean Blvd., Suite 209 Pompano Beach, 33062 Florida, USA*

## Abstract

We present a new theoretical framework for multidimensional image processing using hypercomplex and Clifford algebras. The main goal of the work is to show that commutative hypercomplex algebras and Clifford algebras can be used to solve problems of multi-color image processing and pattern recognition in a natural and effective manner.

**Key words:** Multidimensional image processing, Hypercomplex algebras, Clifford algebras, Pattern recognition

## Synopsis

The term “multicomponent (= multispectral, multicolor) image” is used for an image with more than one component. An RGB image is an example of a color image featuring three separate image components: R (red), G (green), and B (blue). We know that primates and animals with different evolutionary histories have color visual systems of different dimensionality. For example, the human brain uses three channel (RGB) images, reptile and tortoise brains use five channel multicolor images, and shrimps use ten channel multicolor images. We give algebraic models for two general levels (retina and VC) of visual systems using different hypercomplex and Clifford algebras. Our hypotheses are

1. Brains of primates operate with hypercomplex numbers during image processing and recognition.
2. Brains use different algebras on two levels (retina and VC). Multicolor images appear on the retina as functions with values in a multiplet  $mD$  algebra ( $m$ -cycle algebra) where  $m$  is the number of image spectral channels. In particular, RGB-color images as they appear on the

human retina are represented as triplet-valued functions. But multicolor images in an animal's VC are functions with values in Clifford algebra.

3. Visual systems of animals with different evolutionary history use different hypercomplex algebras for color and multicolor image processing.

In the algebraic approach, each pixel is considered not as a multi-dimensional vector, but as a hypercomplex (multi-dimensional) number. For this reason, we assume that the human retina and human VC use 3D hypercomplex (triplet) numbers and 8D Clifford numbers, respectively, to process color (RGB)-images. Note that both these assumptions (vector and hypercomplex natures of multicolor images) are only hypotheses. We have no biological evidence in the form of experiments that would verify that the brain actually uses any of the algebraic properties arising from the structures of vector spaces or Clifford algebras. We only know that animals are able to recognize objects in an invariant manner and to process multicolor images effectively.

It is our aim to show that the use of Clifford algebras fits more naturally to the tasks of recognition of multicolor patterns than does the use of color vector spaces. One can argue that nature has, through evolution, also learned to utilize properties of hypercomplex numbers. Thus, the Visual Cortex might have the ability to operate as a Clifford algebra computing device.

# Structure and Function of Industrial Symbiosis Networks as a Complex Networks

Ana Luković<sup>1</sup>, Srđan Glišović<sup>2</sup>, Vesna Nikolić<sup>3</sup>

<sup>1,2,3</sup>*Faculty of Occupational Safety, University of Niš, Čarnojevića 10A, Serbia*

*E-mail:* <sup>1</sup>julovskiana@live.com, <sup>2</sup>srdjan.glisovic@zrnrfak.ni.ac.rs, <sup>3</sup>vesna.nikolic@zrnrfak.ni.rs

## Abstract

Industrial Symbiosis represents an engagement of traditionally separate industries in a collective approach to competitive advantage, involving the physical exchange of materials, energy, water and by-products. By establishing a collaborative of knowledge, material and energy exchanges among different organizational units, industrial symbiosis networks aim to reduce the intake of virgin materials and lower the production of waste by the industrial sector. Complex industrial system consists of many agents that interacts whit each other at multiple physical or social interfaces – material, energy, information. The main objective of this investigation is to explaine structure and function of industrial symbiosis networks as a complex networks.

**Key words:** Industrial symbiosis, Industrial ecology, Complex networks, by-product exchange, and Eco-Industrial Park.

## Synopsis

Industrial ecology proposes a new organization of industrial activities and processes that mirror the efficient functioning of natural systems, where waste and by-products are subsequently reused and recycled. Industrial ecology seeks to optimize the total materials cycle from virgn material to finished material, to component, to product, to waste product, and to ultimate disposal. Infrostructure of industrial ecology to include: developing and implementing the legal, economic and other incentive systems by which desirable behavior can be promoted, as well as the methodologies, tools, information, resources necessary to define and support such behavior. Industrial symbiosis is a collaborative, multy – industrial approach to improve the environment and economical performance of companies, involving the exchange of waste/by-product as substituties for raw materials [1].

Industrial system is dynamic socio-technical system. Technical network and the actors and bodies that are involved—the social networks—together form an interconnected, complex network. In the technical network, material, energy and information are exchanged over suitable interfaces. Much like ecosystems, industrial system can be seen to constantly evolve because individuals, organizations, and governments decide on, for example, consumption, industrial design and investment, and coordination and regulation, respectively, in response to and in interaction with their respective environments. Development of industrial symbiosis depends on an enabling context of social, informational, technological, economical and political factors [2]. The power to influence this context varies among the agents involved such as the government, businesses or coordinating entities. An industrial symbiosis system, with autonomous industrial actors and bilateral symbiotic relations between them, has been considered as a complex adaptive self-organizing system [3].

Innovation and beneficial collaboration requires a highly flexible network structure that favors the free exchange of information (data collection – basic information about company's operation, including waste handling and by-product reuse) and knowledge about management of raw material, energy and waste. This work describes the structure, function and examples of industrial symbiosis networks as a complex networks.

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# Game Theoretic Strategies That Promote Cooperation in Wireless Networks

Andrej Gajduk<sup>1</sup>, Zoran Utkovski<sup>2</sup>, Lasko Basnarkov<sup>3</sup>, Ljupco Kocarev<sup>4</sup>

<sup>1</sup>*Macedonian Academy of Sciences and Arts, Republic of Macedonia*

<sup>2</sup>*Faculty of Computer Science, University Goce Delcev, Stip, Republic of Macedonia*

<sup>3</sup>*Faculty of Computer Science and Engineering, University SS. Cyril and Methodius, Skopje, Republic of Macedonia*

<sup>4</sup>*BioCircuits Institute, University of California, San Diego, USA*

*E-mail:* <sup>1</sup>agajduk@manu.edu.mk, <sup>2</sup>zoran.utkovski@uni-ulm.de,  
<sup>3</sup>lasko.basnarkov@finki.ukim.mk, <sup>4</sup>lkocarev@ucsd.edu

## Abstract

Energy efficiency is becoming increasingly important in wireless networks. This is emphasized by the emergence of heterogeneous networks which feature devices with low-power capabilities like sensors. We develop a game-theoretic framework to investigate the effect of cooperation on the energy efficiency in wireless networks. Most present approaches address the issue of energy efficiency in communication networks by using complex algorithms to enforce cooperation in the network, followed by extensive signal processing at the network nodes. Instead, we address cooperative communication scenarios which are governed by simple, evolutionary-like, local rules, and do not require strategic complexity of the network nodes. The approach is motivated by recent results in evolutionary biology which suggest that cooperation can emerge in Nature by evolution *i. e.* can be favoured by natural selection, if certain mechanism is at work. As result, we are able to show by experiments that cooperative behaviour can indeed emerge and persist in wireless networks, even if the behaviour of the individual nodes is driven by selfish decision making.

**Key words:** Wireless networks, Evolutionary biology, Energy efficiency, Cooperation

## Synopsis

The provisioning of energy efficient protocols and communication schemes is one of the main challenges in the design of present and future communication networks. Energy efficiency is particularly relevant in emerging heterogeneous networks which include various devices with low-power capabilities, such as sensors and machine-type devices. The study of the fundamental limits of wireless networks suggests that cooperation among the units could contribute to the energy efficiency.



Most of the present approaches which deal with the aspects of cooperative communications, assume that the network nodes act in a pre-defined way, i. e. their behavior is determined by (usually) centralized network rules [1]. Since cooperation is associated with a cost (usually energy) and requires certain signal processing capabilities (computational complexity), this approach may lead to a "cooperation burden" which can be unreasonably high for some network nodes.

Biological systems also exhibit cooperative behaviour. In contrast to man-made communication networks, cooperation in biology is not enforced. Rather it is voluntarily practiced by individual organisms. This shows that cooperation can result from selfish actions and it can be favoured by natural selection. Motivated by these observations we propose a cooperation scheme for wireless networks based on simple local rules which mimic evolutionary principles.

Cooperation is essential to the functioning of a large number of biological systems. However, the presence of cooperation in biological systems has intrigued evolutionary biologists for some time. One would expect that natural selection favors defectors over cooperators. This is true only for are systems where everyone interacts with everyone else equally likely. While studying evolutionary games in structured populations and on graphs, in [4] the authors observe that in structured populations cooperation may emerge given that a certain mechanism is at work. Network reciprocity is one mechanism which explains the emergence of cooperation in structured populations. Network reciprocity is based on the fact that in networks most of the interactions are limited to the neighboring nodes.

In contrast to the previous approaches [3] which enforce cooperation, we are interested in cooperation which emerges as result of the system evolution. An obvious choice for promoting cooperation in wireless networks is network reciprocity. Indeed, if we describe wireless networks as graphs, an analogy can be drawn with populations which are not well mixed. One user can interact only with the nodes which are in its transmission range, thus forming a cluster of potential cooperators.

Several interesting insights appear as consequence of the analysis in this paper. Perhaps, the most important one is that a population of cooperators persists although the nodes decisions are selfish. This in terms results in decreased average energy consumption both for the whole population and for the individual nodes.

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# Self-organization and Creativity in Art

Miloš Milovanović<sup>1</sup>, Milan Rajković<sup>2</sup>

<sup>1</sup>*Mathematical Institute of the Serbian Academy of Sciences and Arts, Belgrade, Serbia*

<sup>2</sup>*University of Belgrade, Institute of Nuclear Sciences Vinča, Belgrade, Serbia*

*E-mail:* <sup>1</sup>milosm@mi.sanu.ac.rs, <sup>2</sup>milanr@vin.bg.ac.rs

## Abstract

Self-organization in the wavelet-tree is used as a criterion for the optimal wavelet basis. The framework of the model is founded on the wavelet-domain hidden Markov model and it is based upon the assumption that statistical complexity is related to information content necessary for maximally accurate prediction of the system's dynamics. The causal states and the wavelet machine ("w-machine") are defined in analogy with the  $\epsilon$ -machine constructed as the unique, minimal, predictive model of the process.

**Key words:** Complex systems, Wavelets, Hidden Markov model, Self-organization,

## Synopsis

In the most general sense, the term self-organization refers to the process that develops over time and which causes the emergence of structures and organized behavior without external influence. A necessary condition for self-organization is an increase in statistical complexity implying that optimal prediction of such a process requires more information. So far, the only methodology that satisfies rigorous requirements for discovering, describing, quantifying and predicting patterns is the so called  $\epsilon$ -machine [1]-[3]. It represents the minimal model for the process that is optimally predictive. Recently, we have developed a "w-machine" (w stands for wavelet) in analogy with the  $\epsilon$ -machine, which is a versatile method for quantifying complexity that at the same time determines the optimal wavelet for capturing and quantifying self-organization of the dynamic system and which also performs superior denoising [3]. It also shares the same properties which used to be unique to the  $\epsilon$ -machine. The framework is founded on the wavelet-domain decomposition and the properties of the wavelet tree (the graph of wavelet coefficients) and statistical properties of the wavelet coefficients. A parametric model for a wavelet tree distribution attributes hidden variable to each node forming a hidden Markov model of the tree. The wavelet decomposition is sparse so that most of the energy is packed into small number of large coefficients, while the remaining large number of small coefficients, carry remaining energy. The tree is considered as a self-organizing system by identifying hidden

states with local causal states. The local statistical complexity of a spatio-temporal process is defined as the Shannon entropy of the local causal state, while the global complexity is the corresponding entropy of the whole tree.

We discuss the relationship of self-organization with creativity in the process of art-work creation and illustrate the application of quantification of self-organization in detecting forgeries of artistic paintings.

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# Universality in Voting Behavior

Marija Mitrović<sup>1</sup>, Arnab Chatterjee<sup>2</sup>, Santo Fortunato<sup>3</sup>

<sup>1</sup>*Scientific Computing Laboratory, Institute of Physics Belgrade, University of Belgrade*

*Pregrevica 118, 11080 Belgrade, Serbia*

<sup>2,3</sup>*Department of Biomedical Engineering and Computational Science, Aalto University School of Science*

*P.O. Box 12200, FI-00076, Finland*

*E-mail:* <sup>1</sup>[marija.mitrovic@ipb.ac.rs](mailto:marija.mitrovic@ipb.ac.rs), <sup>2</sup>[arnab.chatterjee@aalto.fi](mailto:arnab.chatterjee@aalto.fi), <sup>3</sup>[santo.fortunato@aalto.fi](mailto:santo.fortunato@aalto.fi)

## Abstract

Statistical physics provides a conceptual framework for studying large-scale social phenomena. Elections represent a valuable area for quantitative study of human behavior. We analyze the candidate performance in proportional elections with nominative votes within the party list. The study of election data sets from different countries with open-list proportional systems confirms that nations with similar election rules belong to the same universality class. Deviations from this trend are associated with differences in the election rules. Our analysis reveals that voting process is characterized with dynamics that does not depend on the historical, political or economical context where the voters operate.

**Key words:** social systems, universality, voting behavior

## Synopsis

Different physical systems during transitions from one macroscopic state to another exhibit a behavior which is enforced by a few basic features of the individual particles, but independent of all other characteristics. In critical phenomena, like continuous phase transitions this phenomena is known as universality [1]. This simple emergent behavior also occurs when the fundamental constituents are more complex than atoms or molecules. Human societies are characterized by global regularities [2], although their basic constituents are humans, and every individual interacts with limited number of peers, much smaller than the total number of people in the system. Statistical physics provides a conceptual framework for studying this social regularities as collective effects of the interaction among single individuals [3, 4].

Elections are among the largest social phenomena and represent a valuable area for quantitative study of human behavior. The availability of datasets in electronic form results in the fact that elections are one of the most studied social phenomena of the last decade [5]. One of the first, and thus the most, studied feature of distribution of the number of votes of candidates [6, 7, 8]. In their analysis of Brazilian federal and state elections, Costa Filho et. al [6], studied the distribution of the fraction of votes received by candidates, i.e. they analyzed the combined effects of candidate and party popularity on candidates success. They showed that this distribution has a power law behavior with exponent  $-1$  in the central region. Similar results were found for several other countries, see [8] and reference therein. On the other hand, Fortunato and Castellano argued that in order to study competition between candidates, one needs to compare performance of the candidates of the same party [7]. This can be done in proportional elections with open and semi-open lists, where people choose their representatives by voting directly for one or more of them.

Here we show the results of our comprehensive analysis of distribution of candidate's performance. using the results of proportional elections with open and semi-open lists held during several decades in 15 different countries [8]. We conclude that the relative performance, i.e. the ratio between the number of votes of a candidate and the average score of his/her party competitors in a given list has indeed the same distribution for countries with similar voting systems. On the other hand, we don't find the same universal behavior of the distribution of fraction of votes of candidates for different countries, which indicates that party affiliations cannot be neglected.

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# Quantum Structures in Foundations and Applications of Quantum Theory

Jasmina Jeknić-Dugić<sup>1</sup>, Momir Arsenijević<sup>2</sup>, Miroljub Dugić<sup>3</sup>

<sup>1</sup> *University of Niš, Faculty of Science and Mathematics, Višegradska 33, 18000 Niš, Serbia*

<sup>2,3</sup> *University of Kragujevac, Faculty of Science, Radoja Domanovića 12, 34000 Kragujevac, Serbia*

*E-mail:* <sup>1</sup>jjeknic@pmf.ni.ac.rs, <sup>2</sup>fajnman@gmail.com, <sup>3</sup>dugic@open.telekom.rs

## Abstract

Realistic physical systems are composite, i.e. decomposable into subsystems. In classical physics decompositions are often regarded artificial, i.e. as a mathematical artifact. However, in quantum mechanical context, the things are not that simple. In this paper we describe the linear canonical transformations and the induced bipartite structures in different contexts: quantum measurement and decoherence, quantum correlations as resource for the quantum information processing, quantum thermodynamics and certain applications.

**Key words:** Canonical transformations, Tensor-re-factorization of Hilbert space, Quantum measurement and decoherence, Quantum correlations

## Synopsis

Realistic physical systems (classical or quantum) are composite—i.e. decomposable (structured) into constituent subsystems. Transitions between different structures of a composite system are typically described by Linear Canonical Transformations (LCTs). In classical physics, certain structures are regarded artificial, i.e. as mathematical artifacts. However, in quantum mechanical context, the things are more subtle.

LCTs induce re-factorization of the composite system's Hilbert space,  $\mathcal{H}$ . If a composite system,  $C$ , can be decomposed as  $S + E$ , or as  $S' + E'$ , i.e.  $S + E = C = S' + E'$ , the Hilbert state space is re-factorized,  $\mathcal{H}_S \otimes \mathcal{H}_E = \mathcal{H} = \mathcal{H}_{S'} \otimes \mathcal{H}_{E'}$ . Given the composite system is closed (i.e. subject to the unitary Schrödinger law), the system's quantum state is "pure" and unique in every instant of time. However, as it can be shown [1,2], correlation (quantum or classical) in the composite system are not invariant. That is, amount of correlations is different for different structures (decompositions) of the total system  $C$ . So a new rule in the universally valid quantum mechanics is established, the so-called quantum correlations relativity (QCR) [1,2]: The kind and amount

of correlations is not a characteristic of a composite system or of the system's state, but is a characteristic of the composite system's structure. To this end paradigmatic is a description of the hydrogen atom, which is defined as a pair "electron+proton" ( $e + p$ ) as well as "center-of-mass+relative position" ( $CM + R$ ). Solution to the "problem of the hydrogen atom" is obtained for the  $CM + R$  structure (without the atomic spin) in the form of eigenstates  $|\chi\rangle_{CM}|nlm_l\rangle_R$ ;  $n, l, m_l$  denoting the standard quantum numbers. On the other hand, the atomic electron and proton are in interaction via the (classical, nonrelativistic) Coulomb field. Hence there is quantum entanglement for the  $e, p$  pair. So an instantaneous state of the hydrogen atom,  $|\Psi\rangle_{atom}$ :

$$|\chi\rangle_{CM}|nlm_l\rangle_R = |\Psi\rangle_{atom} = \sum_i c_i |i\rangle_e |i\rangle_p, \quad \sum_i |c_i|^2 = 1. \quad (1)$$

Analogous equality applies to the mixed states (density matrices) represented by the self-adjoint, positive, unit-trace linear operators on the system's Hilbert space [1].

As known direct corollaries of QCR we emphasize: (i) parallel occurrence of decoherence for mutually irreducible structures of the standard quantum Brownian motion model [3]; (ii) inconsistency with the Everett interpretation of quantum mechanics [4]; (iii) a limitation of the Nakajima-Zwanzig projection method in open quantum systems theory [5]. Recent proposal of relativity of the concept of thermodynamic equilibrium [6] can be easily recognized as a direct corollary of QCR. So Quantum Correlations Relativity is a topic worth pursuit in both device-independent quantum information theory and applications and quantum thermodynamics.

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# HAARP as a Complex System

**Smiljkovic Kristina**

*Faculty of Occupational Safety, Černojevićeva 10A, 18000 Niš, Serbia*

E-mail: kristina91smiljkovic@gmail.com

## **Abstract**

This paper describes in detail High Frequency Active Auroral Research Program (HAARP). HAARP is a research project that studies the high frequency, stimulates and controls processes in the ionosphere. The purpose Haarp system is to analyze and explore the ionosphere, as well as the phenomenon of the aurora borealis, and radio communication.

**Key words:** system, ionosphere, haarp, antenna

## **Synopsis**

System consisting of a set of elements which interact with each other are referred to as complex system. Interplay between different parts of a complex system of simple systems, and define their internal structure, behavior, and model changes, making them more difficult to understand and manage. Complex systems is characterized by a strong (non-linear) interaction between the parts, complex feedbacks that make it difficult to distinguish is causes from effects and significant spatial and temporal lag, bifurcation and limitations. To understand the interaction of society and nature, man and his environment has to start from the exploration of how these two systems work together, and do not start researching the analysis of these systems by themselves, in isolation. One of the project in complex system is project HAARP (High Frequency Active Auroral Research Program), whose plant was built in Gakoneu, Alaska, USA, in order to study the ionosphere.

The main instruments of Haarp systems are 180 high-frequency antennas organized in 15 columns by 12 rows, which together can transmit 3600 kilowatts of radio waves at frequencies from 2.8 to 10 megahertz (High Rank) which enables extremely sensitive instruments to measure the physical processes that occur in the belt. The signals are generated by the transmitter system over the antennas and is transmitted upward, at an altitude between 70 and 350 km (43 to 217 km) (depending on operating frequency). The signal is partially absorbed in a small volume several tens of kilometers in diameter. The intensity of the high frequency in the ionosphere is less than  $3 \mu\text{V} / \text{cm}^2$ , which is tens of thousands times smaller than the natural electromagnetic radiation reaching the earth from the Sun.



Each antenna element consists of a crossed dipole that can be polarized for linear, ordinary mode (O mode), or extraordinary mode (X-mode), transmission and reception. Each part of the two section crossed dipoles are individually fed from a custom built transmitter, that has been specially designed with very low distortion.

The antennas are arranged in a computer-controlled network, known as a "phased line". The phased array has the capability to focus the radio signals in a more accurate way, without the need for rotating the antennas. Phased array radar at HAARP is used for extremely high frequencies to focus a powerful beam of radio waves at specific locations in the Earth's ionosphere. Extremely high frequency waves, on the other hand, is much shorter than the short waves. Having retained its power over long distances, they are preferred for most communication. For this reason, the high-frequency waves that propagate along said line of sight. When these rays reach the ionosphere, they are deducted from it, and there is a warm-up, or like "cooking" the atmosphere.

Features of the system and process of creations is what we need to understand not just their existence. Therefore, the methodological point of view and the problems of further development in environmental management and the creation of a changed attitude toward nature can be resolved by using compound based on ecological principles.

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# Simplicial complex as Complex System

Milan Rajković<sup>1</sup>, Slobodan Maletić<sup>2</sup>

<sup>1,2</sup> *University of Belgrade, Institute of Nuclear Sciences, Vinča, Belgrade, Serbia*

*E-mail:* <sup>1</sup>milanr@vin.bg.ac.rs, <sup>2</sup>superslobo@vin.bg.ac.rs

## Abstract

We present properties and uses of simplicial complexes in the study of complex systems. We demonstrate that they simplicial complexes offer superior and a more versatile description of complex systems and we demonstrate the new discipline of statistical mechanics o simplicial complexes.

**Key words:** Simplicial complex, Complex networks, Topology, Homology,

## Synopsis

Over the last few years there has been an important development of new ideas and methods in the one of the most exciting areas of mathematics: algebraic topology. As a consequence, the interest in the applications of the methods and results of topology, and algebraic topology in particular, increases steadily. Applied topology is emerging as an important source of ideas, methods and results which diffuse into many areas of research, including the engineering aspects of complex networks ranging from mobile sensor networks [1] to wireless technology [2]. As part of our plan to develop a powerful and versatile method for the analysis of complex systems based on topological methods, we have first combined certain concepts of statistical mechanics of complex networks with the measure on the connected components of the topological space known as the Q-analysis, introduced by Ron Atkin [3], [4]. We have extended this approach [5], [6], through the introduction of persistent homology [7], and by the applications of the higher order combinatorial Laplacian of simplicial complexes [8], [9]. One of the aims of our research program is to promote topological models founded on the structure of the simplicial complexes as the natural setting for social, biological, technological and complex systems arising in physics and other disciplines. We demonstrate that the network phenomena which offers a number of new aspects and results in comparison with the standard graph structure approach. One of the appealing properties of the topological approach is that simplicial complex represents natural extension of the intuitive perception of a Euclidean space since it is formed by gluing together pieces of different dimension. Algebraic topology provides tools for systematic analysis of the process of matching

and relating different pieces to form the entire structure and how each piece contributes to the geometrical representation of the corresponding system. Also, this framework is particularly compelling for capturing group properties which escapes the standard approach based on the graph concept.

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# AHP Conceptual Framework for Ranking the Performance Indicators of Urban Green Lands in Ameliorating Urban Microclimate

Petar Vranic<sup>1</sup>, Miomir Stankovic<sup>2</sup>, Miglena Zhiyanski<sup>3</sup>

<sup>1</sup>PhD student at Faculty of Occupational Safety Nis, Carnojeviceva 10A 18000 Nis, Serbia

<sup>2</sup>Faculty of Occupational Safety Nis, Carnojeviceva 10A 18000 Nis, Serbia

<sup>3</sup>Forest Research Institute - BAS, 132. Kl. Ohridski Blvd 1756 Sofia, Bulgaria

E-mail: <sup>1</sup>petarvvv@gmail.com, <sup>2</sup>miomir.stankovic@gmail.com, <sup>3</sup>zhiyanski@abv.bg

## Abstract

Urban planning and design largely influence the distribution of urban green lands (UGL), and thus, influence ecosystem services provision in urban areas. However, the link between land use management and complex ecosystem service are still not fully utilized. Better understanding of UGL indicators can provide information in decision making for urban planners and designers to prioritise land management interventions. In this paper, we develop a conceptual AHP framework for the systematic ranking of performance UGL indicators, to create link between the common urban land use types of greenery and ecosystem services provision in a spatially understanding manner.

**Key words:** urban green lands, urban planning and design, AHP, ecosystem performance indicators

## Synopsis

Urban systems are built upon complex functional relation between the human-urban-natural agents (people, built environment, vegetation etc.), whose interaction have non-linear behavior. Each urban agent is a complex self-organizing system too, and thus, the urban system is a complex system [1]. Ecological system, and thus UGL, complexity arises from inter-specific and intra-specific interactions among its elements (vegetation species with different structural properties, water, soil etc.), and their interactions with abiotic environment over space and time [2]. Considering that a complex system is any system with numbers of interactive elements usually with hierarchical self-organizing structure [3], than UGL can be considered as complex system, whose properties are defined by numbers of performance indicators [8,9].

Many authors agree on the ability of UGL to ameliorate the urban climate [4,5], thus, urban planning and design process, through land management, are important factors that influence the provision of ecosystem

services. The prior analysis of treatment and planning of UGL in urban planning and design methodologies applied in five largest Serbian cities, pointed to inconsistency in number and definition of qualitative and quantitative indicators for planning and designing of UGL [6]. These indicators make impossible the understanding of the ecosystem service capacities of the existing UGL. Those methodological barriers, following [7], can be seen as cumulative risk, which may support the regressive treatment of UGL and add to an increase of settlements vulnerability to climatic changes. Following that, it is necessary to develop a systemic understanding of ecosystem services of UGL in order to support planning and design decisions, which reflect the main objective of this research.

Given our objectives, we focus our ranking on those ecosystem performance that are most relevant to ameliorate microclimate in urban areas, considering three common urban land use types of greenery: dense forest lands in urban centers (FL), urban gardens and allotments (GAL) and grasslands in urban regions (GL). Indicators of ecosystem services have been investigated by many authors [8,9]. Based on these studies we have selected seventeen key performance indicators. The problem is conceptualized by a hierarchical structure with four levels: the first level is the objective (ranking of performance indicators of UGL), the second level are relevant criteria or greenery type (FL, GAL, GL), the third level are the sub-criteria (carbon sequestration capacity, evapotranspiration, solar radiation reduction, cooling air temperature); and the fourth level are the key indicators (17 in total). The performance indicators will be assessed on the side of forestry experts from Forest Research Institute-BAS from Sofia, and the problem will be solved by their ranking using multi-criteria decision-making AHP model [10].

## Acknowledgment

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# AHP in Individual and Group Decision-making

**Bojan Srdjevic**

*University of Novi Sad, Faculty of Agriculture  
Department of Water Management, Group for Systems Analysis and Decision Making  
Trg D. Obradovica 8, 21000 Novi Sad, Serbia*

*E-mail: bojans@polj.uns.ac.rs*

## **Abstract**

The Analytic Hierarchy Process (AHP) is a widely recognized methodology for solving decision-making problems with multiple criteria and alternatives. The AHP is efficient in use and transparent to one or more decision makers in both instances: (1) while structuring the hierarchy of the problem in hand and judging its elements; and (2) while interpreting the final results represented by the overall weights of alternatives with respect to a stated goal. Thousands of applications world-wide and the reported results of researchers and practitioners show that the AHP is a widely accepted and reliable concept in both the individual and group contexts. In this paper a brief overview of AHP's main features in supporting individual and group decision making is presented, followed by several case study example applications in Serbia.

**Key words:** Decision making, individual and group contexts, Analytic Hierarchy Process (AHP)

## **Synopsis**

The Analytic Hierarchy Process (AHP) [1] belongs to a class of multi-criteria optimization methods. It is a soft computing technique as well as a multi-criteria analysis method, originally developed to support individual decision making. More recently, the AHP is aimed at supporting the decision-making processes in both individual and group contexts. In the latter cases, various aggregation schemes are applicable, e.g., AIJ - aggregation of individual judgments, or AIP - aggregation of individual priorities. In [2] authors argued that AIJ and AIP are philosophically different circumstances, and whether AIJ or AIP should be used depends on whether the group intends to behave as a synergistic unit or as a collection of individuals. More recent published researches demonstrate quality of new aggregation options, e.g. to

identify the locally (at each node of a hierarchy) best individual priorities before the final AHP synthesis is performed. This method is referred to as the MGPS algorithm [3].

It is worth mentioning that there are also various consensus-reaching procedures, such as the CCM - consensus convergent model [4], to mention just one. Interesting reviews of AHP applications can be found in [5, 6].

The decision maker makes judgments more or less consistently, depending not only on his knowledge of the decision problem itself, but also on his ability to remain focused and to ensure that his understanding of the cardinal preferences between the elements will always, or as much as possible, be formalized properly while using a verbal scale or the related numerical ratios. For problems with more hierarchy levels and/or more decision elements (e.g., criteria, sub criteria – sub sub criteria – alternatives) the ability of the decision maker to remain focused and consistent during the decision-making process is highly questionable, because of his/her brain-channel capacity and short-term memory. Experiments have proven that the human brain is limited in both its short-term memory capacity and its discrimination ability (channel capacity) to about seven things. Some people can recall more than seven, some fewer. But only about 11% of the population can recall nine things from their short term memory, still fewer ten things, and so on.

An issue of (in)consistency in decision makers presents the challenge of employing evolution strategies, genetic algorithms, particle swarm optimization and other soft computing techniques and heuristics in deriving priorities from inconsistent (or low consistency) matrices. In a way, a new paradigm aims to provide more efficient solving of the spectrum of decision making problems where individual judgments of decision makers play a leading role.

## Acknowledgment

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# Exceptions in Workflow Systems Used in Medicine and Production

**Dragan Mišić<sup>1</sup>, Nikola Vitković<sup>2</sup>, Miroslav Trajanović<sup>3</sup>**

<sup>1,2,3</sup>*Faculty of Mechanical Engineering, Aleksandra Medvedeva 14, Niš, Serbia*

*E-mail:* <sup>1</sup>misicdr@gmail.com, <sup>2</sup>nvitko@gmail.com, <sup>3</sup>miroslav.trajanovic@gmail.com

## Abstract

This paper describes Workflow management system (WfMS) which tries to connect two different areas: medicine and manufacture. It is a system for choosing, designing and manufacturing osteofixation material. The emphasis is not on the system itself, but on the possibilities related to its adaptation to change of environment.

**Key words:** Workflow management systems, Expert systems, Exceptions

## Synopsis

Workflow systems are systems which enable automatization and monitoring of business processes. These systems are mainly used by banks and insurance companies, where those processes are well organized. WfMS are increasingly used in medicine, as well as in factories.

These systems are based on model which is defined in advance, before the beginning of the process. Due to change of environment during the process execution, it is often necessary to change process model. The challenge to researchers is to provide that systems keep functioning in cases when the process model is changed [1]. On that occasion, it is necessary to change the process definition, but at the same time to continue with execution in accordance with new definition. Problems that also occur are activities of old process which are already executed (compensation might be needed).

Commercial WfMS usually deal with mentioned problems by aborting the process and starting a new instance, based on new and changed definition. Scientific WfMS try to solve the problem by adapting the process [2,3]. That means that process continues according to the new definition, and new ones eventually compensate executed activities. There are also approaches where process model is not defined so strictly. Instead, before starting new activities, met conditions are checked and based on this it is decided which the next activity is [4].



Problems with existing WfMS and their adjustment to new situation arise from the fact that for change of a process it needs to reach conclusion based on new facts. This reasoning is characteristic for people, but not for computer software. In an attempt to make a system more intelligent, our WfMS [5,6] is connected to expert system, which tries to solve the problem and enable process execution to be continued.

Connection between WfMS and expert system is demonstrated on the example of process of choosing, designing and manufacturing osteofixation material. It is about different types of fixators or scaffolds which help with recovery of a patient in case of bone fracture. The process is managed by virtual enterprise, which should deliver osteofixation material that is adapted to patient's needs.

Participants of the process are doctors, engineers and personnel from the enterprise which should manufacture and deliver the material that doctors request. We are talking about a process in which various deviations from defined scenario can arise. In this case, solving a problem is described on the example of exception which occur when it is not possible to get an accurate CT image of damaged leg bone, due to conditions of fracture or poor health of a patient. If this is the case, expert system will recommend either taking an image of sound leg or, if that is not possible, measuring the distance between certain points on sound leg or a broken one, and choosing the fixator based on measured values.

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# Project Management Using Bayesian Belief Networks

Natasa Glisovic

*Department for Mathematical Sciences, State University of Novi Pazar, Serbia.*

*E-mail: natasaglisovic@gmail.com*

## Abstract

Bayes' Theorem provides the project manager information in the form of probabilities about the performance of one project activity when it is conditioned upon the performance of another. Bayesian networks have gained importance while dealing with uncertainty and probabilistic reasoning. BNs are an ideal decision support tool for a wide range of problems and have been applied successfully in a large number of different settings such as medical diagnosis, credit application evaluation, software troubleshooting, safety and risk evaluation. Project scheduling inevitably involves uncertainty. The basic inputs (i.e. time, cost and resources for each activity) are not deterministic and are affected by various sources of uncertainty. Moreover, there is a causal relationship between these uncertainty sources and project parameters; this causality is not modelled in current state-of-the-art project planning techniques (such as simulation techniques). Most projects usually have a pre-defined time (i.e. deadline), cost (i.e. budget) and quality (i.e. requirement and specifications) which are discussed and agreed in the project contract. This paper introduces an approach, using Bayesian network modelling, that addresses both uncertainty and causality in project scheduling. The model presented empowers the traditional Critical Path Method (CPM) to handle uncertainty and also provides explanatory analysis to elicit, represent, and manage different sources of uncertainty in project planning. Validity of proposed approach is tested on a project of franchising implementation in PE of PTT Communications "Srbija".

**Key words:** project management, Bayesian networks, Critical Path Method, time cost tradeoff, project scheduling.

## Synopsis

Project management is the ability to plan and monitor their activities and progress within the cost and time and performance, with the goal of competitiveness. (Krajewski and Ritzman, 2005) Since the

1950s, critical path method (CPM) has been demonstrated to be a useful tool in managing projects in an efficient manner to meet the challenge of managing complicated projects (Kelly, 1961; Siemens, 1971; Taha, 2003). In the past, the scheduling of a project (over time) was done with little planning. The best-known “planning” tool then was the Gantt bar chart, which specifies the start and finish times for each activity on a horizontal time scale. Its disadvantage is that the interdependency between the different activities (which mainly controls the progress of the project) cannot be determined from the bar chart. The growing complexities of today’s projects have demanded more systematic and more effective planning techniques with the objectives of optimizing the efficiency of executing the project. Efficiency here implies affecting the utmost reduction in the time required to complete the project while accounting for the economic feasibility of using available resources (Taha [5]). Shortening the duration of the project may be the result of delays or for any reason, the need for an early ending. The project manager, knowing the critical path, with the help of your team can use several techniques to shorten project. Crashing is a technique for making cost and schedule time trade-offs to obtain the greatest amount of time compression for the least incremental cost. This technique is also known as time cost tradeoffs.

The Bayesian networks provide a method for representing the relations between the variables (Bayesian nodes in the network) even when these relations contain uncertainty. The analysis included only risk time. The results indicate that the project will be completed without delay with a probability of 84%. Also, by analyzing the Bayesian network it is necessary to examine more closely all the uncertainties and risks of this activity.

If the analysis includes the cost and the risk of the project activities based on the Bayesian networks for such a formulated problem of the probability of the completed project on time is reduced from 85% to 82.6%. The uncertainty of the realization of certain activities affect the uncertainty of the realization of the project as a whole.

In this paper, we propose the Bayesian networks approach that can include the risk, the uncertainty and the causality in the allocation of the project activities. The approach was tested on the example of the introduction of franchise in the Post of Serbia.

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# On Modelling Bivariate Time Series Of Counts

Popović Predrag<sup>1</sup>, Nastić Aleksandar<sup>2</sup>, Ristić Miroslav<sup>3</sup>

<sup>1</sup>*Faculty of Civil Engineering and Architecture, University of Niš, Serbia*

<sup>2,3</sup>*Faculty of Sciences and Mathematics, University of Niš, Serbia*

*E-mail:* <sup>1</sup>popovicpredrag@yahoo.com, <sup>2</sup>anastic78@gmail.com, <sup>3</sup>miristic72@gmail.com

## Abstract

Bivariate time series of counts are discussed. Structure of these models are similar to the standard autoregressive (AR) models of order one. AR-recursion is achieved by using thinning operator. AR models for time series of counts are composed of two processes: survival part and innovation part. Models discussed in this article have the survival part based on binomial as well as negative binomial thinning operators. Innovation components that figures in these models are defined in two ways. The first approach defines innovation processes after definition of marginal distribution of observed processes and with the second approach innovation processes are constructed first. Poisson, Geometric and Negative binomial distributions are considered as marginal distribution of the processes.

**Key words:** integer valued autoregressive models, time series of counts, binomial thinning, negative binomial thinning, poisson distribution, geometric distribution, negative binomial distribution

## Synopsis

Nonnegative time series of counts arise in many fields of science. Like in telecommunications number of received messages, in biology number of species, in finance number of transactions, in criminology number of committed crimes etc. Capturing and modelling their behavior are scope of interest for many researchers. Integer valued autoregressive models were introduced by [3] and [1]. When two or more series interact arises the need for multivariate models. Multivariate time series of counts are introduced by [2] where properties and existence of the model are discussed. We give insight into modelling bivariate integer valued autoregressive time series (BINAR). These models are composed of survival and innovation components. Construction of survival component is based on binomial or negative binomial thinning operator. While the first one is suitable for series where the survival process is a short memory process the other one is suitable when objects of a process are able to interact and produce new ones. Binomial thinning operator, denoted by  $\circ$ , is defined as  $\alpha \circ X = \sum_{i=1}^X B_i$  where  $\{B_i\}$  is a sequence of iid Bernoulli random variables with parameter  $\alpha$ . Negative binomial thinning operator, denoted with  $*$ , is defined as  $\alpha * X = \sum_{i=1}^X G_i$  where  $\{G_i\}$  is a sequence of iid Geometric random variables with parameter  $\frac{\alpha}{1+\alpha}$ .

In this talk we discuss BINAR models of the form

$$\mathbf{Z}_n = \mathbf{A}_n * \mathbf{Z}_{n-1} + \mathbf{e}_n$$

where  $\mathbf{Z}_n$  and  $\mathbf{e}_n$  are two-dimensional random vectors and  $\mathbf{A}_n$  is  $2 \times 2$  matrix whose elements are random variables. Different thinning operators are considered.

There are two approaches for construction of BINAR models. With the first one, marginal distribution of the processes are pre assumed and distribution of innovation processes are derived to fulfill stationarity. The second approach starts from definition of innovation processes distribution which implies a distribution of the processes under study. BINAR model defined by the first approach was introduced by [6]. This model has geometric marginals where survival components evolve under negative binomial thinning operator. The second approach for constructing BINAR model was introduced in [4] and [5]. Both models have survival parts that evolve under binomial thinning operator. We give a brief insight into properties of all these models, suggest methods for parameter estimation, discuss goodness of fit on a real data and analyze forecasting residuals.

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# E-Learning Computing Grid as Complex System

Ivan S. Živković

*Mathematical Institute of the Serbian Academy of Sciences and Arts,*

*Kneza Mihaila 36, 11001 Beograd, Serbia*

*E-mail: zivkovic.ivan83@gmail.com*

## Abstract

E-Learning can be defined as the use of new multimedia technologies and the Internet to improve the quality of learning by facilitating access to resources and services, as well as remote exchanges and collaboration. At present, most E-Learning environment architectures use single computers or servers as their structural foundations. But, as work loads increase, software and hardware must be updated or renewed. By employing Computing Grid infrastructure the need for purchasing costly high-level servers and other equipment can be eliminated. By sharing the processing power and storage space of many learning devices, E-Learning could address tasks such as medical image processing, which require significant amounts of computing power. Sharing resources would be of particular benefit for mobile learning, where learners use portable devices with limited memory and processing power. The system can be represented with two layers which communicates to each other through intelligent units.

**Key words:** E-Learning, Computing Grid, Complex Computer System, Intelligent Hub, Scale Free Network

## Synopsis

Grid Computing is a set of distributed computing resources available over a Local Area Network (LAN) or Wide Area Network (WAN) that become visible to an end user or application as one huge virtual computing system. The objective is to create virtual dynamic organizations through secure, unlimited power, information access, synchronized resource-sharing among users, institutions and resources.

E-Learning Grid in turn represents the amalgamation of Grid Computing and E-Learning in which of Grid Computing functionalities are incorporated into E-Learning systems. E-Learning Grid is a collection of computational resources on demand to match computational needs through a sort of generic service matchmaking. More precisely implementation of a network grid supporting an E-Learning infrastructure embraces many of the requirements for

exchanging information in the educational and research fields. These networks have property that number of links  $k$  originating from a given node exhibits a power law distribution i.e. only few nodes (called *hubs*) have a large number of links. These networks are called scale-free and they are located in between the range of random and completely regular wired networks.

E-Learning Grid as a whole can be considered as a complex system (more properly a Complex Computer System) whose aim is to provide problem solutions to the end-user. From the end-user (e.g. learner) point of view, Grids may provide the following types of services: Computational Services [1], Data services [2], and Application services [3]. Only regarding the grid from a multi-disciplinary point of view can help us understand the behavior of these systems. In this paper we will show how an E-Learning Grid can be represented through a model as a complex system that can be used for simulations. E-Learning Grid can be represented as two-layer model: physical layer and logical layer. The first layer is the physical structure of the resource grid itself. The second layer includes all the information exchange that has to be arranged to coordinate resource sharing and problem solving in dynamic, multi-institutional virtual organizations. To model interaction between the layers, we introduce the concept of *intelligent hub*, specially designed generic units. Those units can:

- have a communication interfaces consisting of functions of the logical layer
- monitor the physical layer (physical resources)
- perform data processing, job scheduling, and evaluation
- handle any interactions between layers

In this way, logical layer can be viewed in the sense of the interactions between intelligent hubs, which represents nodes in this complex system.

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# Pedagogical Issues in the Design of an E-learning System

Vesna Nikolic<sup>1</sup>, Jelena Ranitovic<sup>2</sup>

<sup>1</sup>*Faculty of Occupational Safety, University of Nis, Carnojevica 10A, 18000 Nis, Serbia*

<sup>2</sup>*Faculty of Philosophy, University of Novi Sad, Dr Zorana Djindjica 2, 21000 Novi Sad, Serbia*

*E-mail:* <sup>1</sup>vesnik08@gmail.com, <sup>2</sup>jelena.ranitovic@sipcoms.org

## Abstract

A complex e-learning system includes numerous pedagogical and didactic-methodological issues related to teaching and learning. Pedagogical-didactic system, being a subsystem of the complex e-learning system, contains a number of dilemmas and unsolved problems which are all related to the analysis of the e-learning system as it plays an important role in the process of knowledge acquisition. Some of the problems deal with didactic principles of e-learning, selection of educational resources and editing tools, preparation and formatting of learning and teaching materials, both electronic and printed. Internal organization of e-learning, its learning methods, forms, and techniques, as well as structure and articulation of an individual lesson are being discussed as potential problems, too. Special teaching methods and tutoring, evaluation and assessment are equally important in e-learning. Basic pedagogical issues are related to content analysis, audience analysis, goal analysis, media analysis, approaches to instructional design and learning strategies. All decisions related to the complex e-learning system design can be implemented on the basis of pedagogical analyses presented and defined in this paper.

**Key words:** pedagogical issues, complex e-learning system, analysis

## Synopsis

Selecting e-learning content depends on the learning objectives. The content refers to a particular subject matter within domain knowledge of a specific e-learning course. Content analysis is carried out by the subject matter experts. It may be helpful if the program creator determines which aspects of the content domain are appropriate for e-learning and which are appropriate for hybrid courses [1,2]. It is of great significance to determine stability of the course content, its statics and dynamics, and to identify content types. Audience analysis should provide plenty of important information to create e-learning activities. Keeping in mind that e-learners come from various socio-cultural environments, information



about their knowledge and skills, learning styles, personal and social characteristics, needs and interests represent the most essential part of the audience analysis [3,4]. Therefore, it is advisable to use different data collection techniques in these cases, such as surveys, interviews, observations, documentation reviews, etc.

Goal analysis helps to identify and clarify objectives of the e-learning projects. Firstly, e-learners should have clear goals and objectives, and then they have to try to find reasonable ways to achieve them. Media analysis shows how systems of course delivery in e-learning can facilitate learning [5,6]. In this regard, designers should be familiar with various options related to delivery mediums. Media choice in e-learning significantly depends on the content identification and analysis, whether it is static, dynamic or variable content in question. Creators should analyze characteristics of each delivery mode, i.e. Internet, CD-ROM, DVD, face-to-face learning, printed resources based on books, articles and manuals, in order to determine their applicability in e-learning. Design approach to e-learning activities depends on types of domain knowledge or e-learning content. Pedagogical philosophy of the overall course design is influenced by well-structured content or ill-structured content- instructivist vs. constructivist approach. In order to design successful e-learning course, the control of e-learning activities, both student-directed and program-directed control is very important.

A variety of teaching methods and strategies can be used in organization of e-learning activities. Therefore, acquiring knowledge using complex learning system is described in this paper. This learning system is extremely elaborative and complicated. It requires teamwork approach, respectively in the stages of preparation, execution, and evaluation. Pedagogical dimension of e-learning involves a combination of content analysis, analysis of e-learner needs, and goal analysis. These dimensions influence both design and strategies of e-learning.

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# Model for E-learning Quality Indicators Evaluation

**Suzana Savić<sup>1</sup>, Goran Janacković<sup>2</sup>, Radojka Krneta<sup>3</sup>, Danijela Milosevic<sup>4</sup>,**

<sup>1,2</sup>*University of Niš, Faculty of Occupational Safety, Černojevića 10a, Niš, Serbia*

<sup>3,4</sup>*University of Kragujevac, Technical Faculty Čačak, Svetog Save 65, Čačak, Serbia*

*E-mail:* <sup>1</sup>suzana.savic@znr fak.ni.ac.rs, <sup>2</sup>goran.janackovic@znr fak.ni.ac.rs, <sup>3</sup>rkrneta@ftn.kg.ac.rs,  
<sup>4</sup>danijela.milosevic@ftn.kg.ac.rs,

## Abstract

E-learning is becoming increasingly important for the competitive advantage of higher education institutions. The paper examines e-learning quality criteria and indicators and presents a model for e-learning quality evaluation based on the method of Analytic Hierarchy Process and SEVAQ self-evaluation.

**Key words:** quality assurance, self-evaluation, e-learning, SEVAQ+, AHP

## Summary

Quality assurance is a continuous process of evaluating the quality of a higher education system, institutions, or programs. There is usually a distinction between internal quality and external quality assurance. Internal quality assurance consists of inter-institutional practices in the context of monitoring and improving the quality of higher education. External quality assurance defines inter-institutional schemes that are used to assure the quality of higher education institutions and programs [1]. Development of the Distance Learning (DL) Quality Assurance (QA) system required firstly a firm set of responsibilities and activities performed by the higher educational institution. QA of DL has to integrate into the overall institutional QA system. There is no unified approach to QA of DL adopted from relevant European institutions. Still, there have been several initiatives in the past to address quality in ICT-based learning or e-learning [2].

SEVAQ+ represents combination of tool and methodology for the self-evaluation of quality in Technology-Enhanced Learning. It merges two widely recognized evaluation and quality approaches –

the Kirkpatrick and EFQM (European Foundation for Quality Management) models. SEVAQ+ enables three domains of the evaluation from the EFQM model: the resources, the processes (activities) and the results. Each domain contains more criteria, which consist of more indicators. The paper [2] defined eight criteria and twenty indicators as follows: information provided (availability of learning opportunities, course prospectus); learning materials (availability, pedagogical aspects of learning content, coherence with promises, advanced concerns about the quality of resources for the learner); general services offered to the learner (organization services and administration); eLearning activities (time management, navigation and resource options, training approach, personalization, collaboration and self-study); pedagogical support (group learning support); knowledge assessment (assessment process design, assessment process management); knowledge increase (levels of overall knowledge outcomes); performance of learning outcomes (learner's perspective); motivation to learn effectively (awareness of learning preferences, learning management, self-motivation).

SEVAQ+ has a hierarchical organization (domain, criteria, and indicators), and survey results are presented in the form of radar diagram that contains individual assessment indicators and average score for all indicators. The assumption is that all indicators are equally important. However, it is likely that there are different perceptions of indicators with different meanings. Therefore, it is necessary to combine these results with ranks of indicators that can be identified by means of AHP method that supports SEVAQ+ hierarchical organization [3, 4]. It would give a better insight into the performance quality and indicate the actions to be taken in order to ensure quality, and that these actions are not primarily aimed to improve the performance indicators with the lowest scores. Evaluation of e-learning at the master study program was done at the Technical Faculty, University of Kragujevac. Currently, there is a survey on students' preferences about the domain, criteria and indicators of the quality of e-learning.

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# Multi-criteria Decision Analysis with Stochastic Dominance Approach

Aleksandar Janjić<sup>1</sup>, Miomir Stanković<sup>2</sup>

<sup>1</sup>University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, 18000 Niš, Serbia

<sup>2</sup>Faculty of Occupational Safety Nis, Carnojeviceva 10A 18000 Nis, Serbia

E-mail: aleksandar.janjic@elfak.ni.ac.rs, miomir.stankovic@gmail.com

## Abstract

This paper considers a discrete stochastic multiple criteria decision making problem with different types of criteria tradeoffs. The proposed methodology is based on numerical convolution of criteria probability distribution functions, according to different types of criteria aggregation. After the new, aggregated probability distribution is built for every alternative. They are ranked by the stochastic dominance approach. The new methodology allows the usage of compensatory aggregation, which is more suitable for conflicting criteria or the human aggregation behaviour.

**Key words:** Convolution, Multicriteria decision making, OWA, Stochastic dominance

## Synopsis

Stochastic multiple criteria decision making (SMCDM) refers to the problem of selecting alternatives associated with multiple attributes/criteria, where consequences of alternatives with respect to criteria are in the form of random variables. We consider a decision problem consisting of  $n$  alternatives denoted by  $a_i$ ,  $i \in \{1, \dots, n\}$ , each evaluated on  $m$  criteria denoted by  $c_j$ ,  $j \in \{1, \dots, m\}$ . Let  $e_{ij}$  be the evaluation of  $a_i$  in terms of criterion  $c_j$ , according to some suitable performance measure. Our concern is with decision making situations in which the values of  $e_{ij}$  for each  $i$  are not known with certainty for all  $j$ . This formulation is known as ACE (or AEE) model (Alternatives, Attributes/Criteria, and Evaluators). In decision analysis based on probabilities, for each alternative  $a_i$ , the  $e_{ij}$  are viewed as random variables with an associated ( $m$ -dimensional) multivariate probability distribution function  $F_i$ .

There are three main methods to solve SMCDM problem: expected value variance (EV) method [1], stochastic dominance (SD) method [2] and stochastic multi-objective acceptability analysis (SMAA) method [3,4,5]. Some of the issues like the usage of more complex utility functions together with the

correlation between attributes remained neglected. A new methodology based on stochastic dominance of aggregated probability distribution functions of alternatives is presented. The aggregated probability distribution is formed by the numerical convolution of marginal probability distributions. The stochastic dominance is evaluated only on aggregated probability functions, instead of pairwise comparison of alternatives for individual criterion, which makes the application of methodology more practical. The problem is presented as a three step decision problem. In the first stage, the risk attitude of the evaluator is taken into account by the appropriate form of multiplicative utility function or adequate OWA operator. In the rest of the decision process, after the convolution of probability distributions, the decision maker chooses an alternative according to his risk attitude using the appropriate stochastic dominance rule.

The following three types of aggregation are used most commonly in decision making: conjunctive, disjunctive and compensatory aggregation of criteria. Conjunctive aggregation of criteria implies simultaneous satisfaction of all decision criteria, while the disjunctive aggregation implies full compensation amongst the criteria. The compensatory aggregation is more suitable for conflicting criteria or the human aggregation behavior. The application of this methodology is not restricted to probability distributions represented as vectors, because of possible discretization of available CDF. Together with the multiple uncertainties of evaluations and weighting factors, this discretization will be the focus of further researches of the possible application of this methodology.

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